Group delay distortions in electroacoustical systems

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We report on measurements of group delay distortions introduced into electroacoustical transmission by the fact that earphones and loudspeakers are not necessarily minimum phase systems. Psychoacoustical tests show further that the measured distortions can approach the magnitude of the threshold of perceptibility, but in most cases will be well below this value.

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INTRODUCTION

Generally, electroacoustic audio transmission systems are built up by linear, time-invariant (LTI) subsystems (amplifiers, delay lines, loudspeakers, earphones, etc.) whose transfer properties are completely characterized by their Dirac impulse response $h(t)$ or by their complex transfer function $H(\omega)$, which is the Fourier transform of $h(t)$:

$$H(\omega)=\int_{-\infty}^{+\infty} h(t)e^{-j\omega t}dt\,. \qquad (1)$$

Splitting up the transfer function $H(\omega)$ into magnitude and phase according to

$$H(\omega)=|H(\omega)|e^{-jb(\omega)} \qquad (2)$$

one gets the "amplitude response" $|H(\omega)|$ or the "attenuation ratio" $a(\omega)$, respectively, given by

$$a(\omega)=-\ln|H(\omega)| \qquad (3)$$

and the "phase factor" $b(\omega)$.

Both, the amplitude response $|H(\omega)|$ [or attenuation ratio $a(\omega)$] and the phase factor $b(\omega)$ of a system depend characteristically on the angular frequency ω; just so does the derivative of the phase factor $b(\omega)$ with respect to ω, the so-called "group delay,"

$$\tau_g(\omega)=\frac{d}{d\omega}b(\omega)\,. \qquad (4)$$

Especially Eqs. (3) and (4) describe the fact that the single spectral components of an input signal arrive at the output of a general LTI system not only with different attenuations but also at different times, i.e., the output signal (with respect to the input signal) is distorted. In this connection the terms "attenuation distortion" and "group delay distortion" are usual.

Furthermore, it should be noted that a general LTI system characterized by its transfer function $H(\omega)$ can be split into two subsystems, namely into a "minimum-phase system" and into an "all-pass system", characterized by $H_M(\omega)$ and $H_A(\omega)$, respectively, as shown in Fig. 1.

A minimum-phase system is a system whose phase factor $b_M(\omega)$ is completely and uniquely defined by its attenuation ratio $a_M(\omega)$ (see formula 11), i.e., a given attenuation ratio characteristic determines the phase factor or group delay characteristics and vice versa.

An all-pass system is a system whose attenuation ratio specifies

$$a_A(\omega)=0 \text{ for all } \omega \text{ with } -\infty<\omega<+\omega\,, \qquad (5)$$

whereas its phase factor $b_A(\omega)$ or its group delay $\tau_{gA}(\omega)$ is adjustable by suitable realization methods.

Observing the Eqs. (4), (5), and

$$H(\omega)=H_M(\omega)H_A(\omega)\,, \qquad (6)$$

the attenuation ratio $a(\omega)$, the phase factor $b(\omega)$, and the group delay $\tau_g(\omega)$ of a general LTI system can be written in the form

$$a(\omega)=a_M(\omega)\,, \qquad (7)$$

$$b(\omega)=b_M(\omega)+b_A(\omega)\,, \qquad (8)$$

$$\tau_g(\omega)=\tau_{gM}(\omega)+\tau_{gA}(\omega)\,. \qquad (9)$$

As can be seen from the above, the realization of a general LTI system with given attenuation ratio $a(\omega)$ and given group delay $\tau_g(\omega)$ must take place in the following manner:

First, the attenuation ratio $a(\omega)$ is realized by a minimum-phase system, whereby $\tau_{gM}(\omega)$ is determined [see Eq. (9)]. Thereafter the difference

$$\tau_{gA}(\omega)=\tau_g(\omega)-\tau_{gM}(\omega) \qquad (10)$$

has to be realized by means of an additional all pass.

The design of equipment for electroacoustical trans-

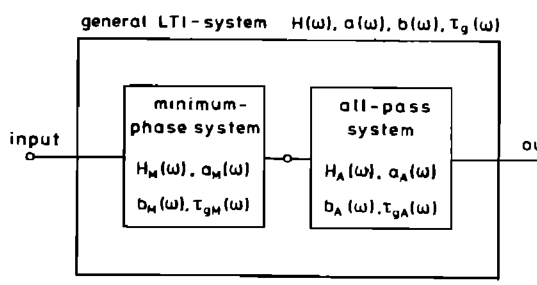

FIG. 1. General LTI-System represented as a series connection of a minimum-phase system and an all-pass system.

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mission of audio often includes the task of correction of attenuation and group delay distortions in such a way that given attenuation ratio and group delay curves are achieved. This is obtained by correcting LTI systems, called "audio equalizers". However, it is often found that the usual audio equalizers allow the required attenuation ratio of the system to be achieved, but not the desired group delay curve.

This is caused by the fact that the usual equalizers are minimum-phase systems, whereas in general the systems to be corrected are not minimum-phase systems especially if they include electroacoustical transducers such as earphones and loudspeakers. Other authors (e.g., Heyser, 1969; Zwicker, 1972) have already stated that loudspeakers are not necessarily minimum-phase systems.

A system which is not a minimum-phase system shows additional all-pass characteristics and so causes additional group delay distortions which cannot be corrected by minimum-phase equalizers alone. To complete group delay correction in these cases, one needs—as it has been explained above—all-pass networks, which means addtional expense.

The experiments reported on in this article have been carried out to obtain pilot data for the answers to the following questions:

(a) What values of group delay distortions caused by all-pass characteristics can be expected in ordinary earphones and loudspeakers?

(b) Are those group delay distortions audible, and, therefore, to what extent must they be corrected?

Literature data concerning question (a) are rare. Heyser (1969, 1971) measured time delay distortions of loudspeakers but did not plot his data in terms of group delay, because group delays are sometimes difficult to interpret physically (e.g., when negative values of group delay occur, which is possible). In this article we restrict ourselves to group delays caused by additional all-pass characteristics. These are always positive and can be interpreted as additional delays of the signal or its parts.

As to question (b), it is known that the ear can detect phase distortion. The literature dealing with this question (e.g., Mathes and Miller, 1947; Schroeder, 1959; Craig and Jeffress, 1962; Goldstein, 1967; Plomb and Steeneken, 1969; Stodolsky, 1970; Hansen and Madsen, 1974; Fleischer, 1975) indicates that our hearing system is especially sensitive to phase distortion within critical bands. However, interpretation of these data in terms of group delay perceptibility seems to be difficult.

Perceptibility of group delay is dealt with in the articles of Bürck, Kotowski, and Lichte (1935); Hilliard (1964); Zwicker and Feldtkeller (1967); Patterson and Green (1970); Klipsch (1972); and Laws and Blauert (1973). Thresholds of perceptibility between 1 ms and more than 10 ms have been measured for group delays. The measured thresholds strongly depend on the shape of the group delay curve, the nature of the test signal, the conditions of stimulus presentation and the state of training of the subjects.

To measure data for practical use, it turned out to be necessary to measure first some typical all-pass distortions as they actually occur in loudspeakers and earphones. Thereafter such distortions were to be simulated to test their perceptibility.

I. EXAMINATION OF ALL-PASS CONTENTS OF LOUDSPEAKERS AND EARPHONES

As we reported in detail in a prior paper (Blauert, Laws, and Platte, 1974) we measure the transfer function of electroacoustical systems by an impulse method. Thereby we obtain frequency functions of attenuation ratio and group delay, computed for every one-sixth octave. On the basis of these data the all-pass contents of the systems under test should be examined.

The following relation exists between the phase factor and the attenuation ratio of a minimum-phase system (Hilbert transform):

$$b(\omega_0)=\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{a(\omega)}{\omega-\omega_0}d\omega\,. \tag{11}$$

Using the fact that $a(\omega)$ is an even function, this can be converted to

$$b(\omega_0)=\frac{2\omega_0}{\pi}\int_0^{\infty}\frac{a(\omega)}{\omega^2-\omega_0^2}d\omega\,. \tag{12}$$

However, Eqs. (11) and (12) are poorly adapted to numerical calculations because they lead to difficulties of convergence.

Therefore a partial integration $uv'dx=uv-vu'dx$ with $u=a(\omega)$ and $v'=(\omega^2-\omega_0^2)^{-1}$ is carried out. One obtains

$$b(\omega_0)=\frac{1}{\pi}\int_0^{\infty}\frac{d[a(\omega)]}{d\omega}\ln\left|\frac{\omega+\omega_0}{\omega-\omega_0}\right|d\omega\,. \tag{13}$$

Since we are not interested in phase but in the group delay of a minimum-phase system, the above expression is differentiated with respect to ω_0

$$\tau_g(\omega)=\frac{d[b(\omega_0)]}{d\omega_0}=\frac{d}{d\omega_0}\left\{\frac{1}{\pi}\int_0^{\infty}\frac{d[a(\omega)]}{d\omega}\ln\left|\frac{\omega+\omega_0}{\omega-\omega_0}\right|d\omega\right\}\,. \tag{14}$$

After exchanging the order of differentiation and integration and some intermediate calculations one gets

$$\tau_g(\omega_0)=\frac{1}{\pi}\int_0^{\infty}\frac{d[a(\omega)]}{d\omega}\,\frac{2\omega}{\omega^2-\omega_0^2}d\omega\,. \tag{15}$$

Rewriting with

$$\frac{d(a/N_p)}{d(lg\,f/\mathrm{Hz})}=-\frac{\ln 10}{20}\,\frac{d(A/\mathrm{dB})}{d(lg\,f/\mathrm{Hz})} \tag{16}$$

and

$$\frac{2\omega}{\omega^2-\omega_0^2}=-\frac{1}{\pi f_0}\,\frac{f/f_0}{1-(f/f_0)^2} \tag{17}$$

leads to

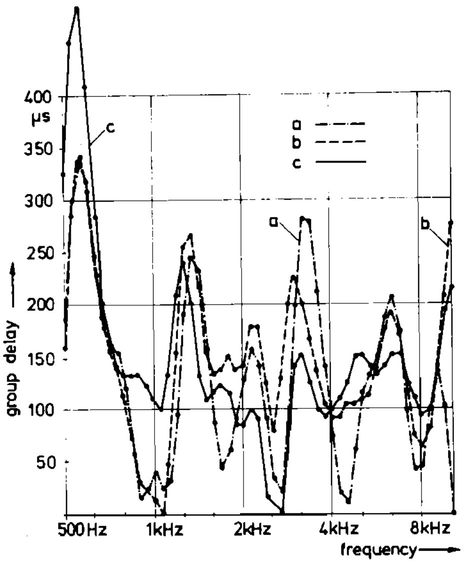


FIG. 2. Group delay caused by all-pass contents: (a) open earphone (Sennheiser HD 414), pickup point: ear drum, (b) open earphone (Sennheiser HD 414), pickup point: entrance of ear canal, and (c) circumaural earphone (Beyer DT 48), pickup point: entrance of ear canal. The same subject was used to obtain all three measurements.

$$\frac{\tau_g(f_0)}{\text{ms}}=\frac{(\ln 10)10^3}{20\pi^2}\,\frac{1}{f_0/\text{Hz}}\times\int_{f=0}^{f=\infty}\frac{d(A/\text{dB})}{d(\lg f/\text{Hz})}\,\frac{f/f_0}{1-(f/f_0)^2}\,d(\lg f/\text{Hz})\,. \quad (18)$$

This is the basic expression used in the numerical calculations. By its use the group delay curve $\tau_g(f_0)$ of that hypothetical minimum-phase system is computed which has the same attenuation characteristic as the system under test. The group delay curve of this (hypothetical) minimum-phase system is then subtracted from the group delay curve of the system under test.

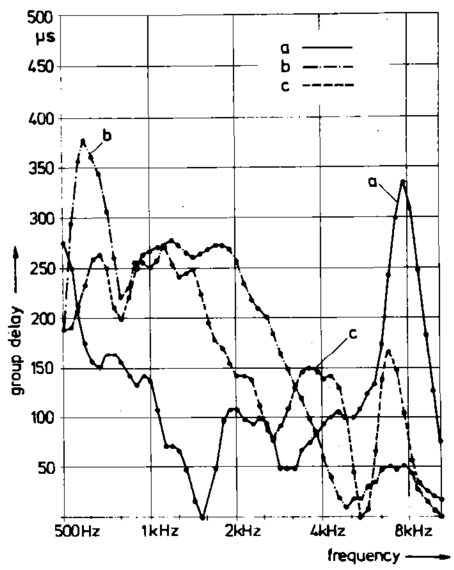


FIG. 3. Group delay caused by all-pass contents: (a) Philips 9710 M loudspeaker in 16 dm^3 closed box, (b) Heco B 180/M closed box (approximately 14 dm^3), (c) Isophon KSB 12/8 closed box (approximately 6 dm^3). Pickup point on the axis, distance 1 m, anechoic chamber.

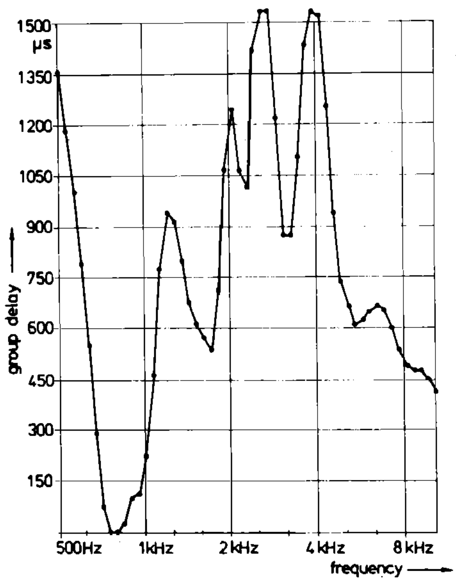


FIG. 4. Group delay caused by all-pass contents: noncommercial bass reflex box with exponential horn (approximately 450 dm^3). Pickup point on the axis, distance 3 m, anechoic chamber.

In this way we obtain the group delay curves shown in Figs. 2–4. They show the group delays caused by the all-pass content of each of the systems identified in the captions. It should be pointed out that the all-pass components can be synthesized from the following network elements: First-order all-pass filters and/or second-order all-pass filters and/or transmission lines (with and/or without losses). The inaccuracy of the results, caused by measurement and computational errors is estimated to be about ± 25 μs. This estimate was verified by the use of an analog group delay meter after Nyquist and Brand.

When plotting the group delay curves it was kept in mind that the additional group delays caused by all-pass characteristics must always be positive. Additive terms of group delay which are constant for all frequencies (as, e.g., those produced when the signal travels a distance in the air) are not included in the plots.

In Fig. 2 the all-pass component of the group delay of a pair of earphones is shown. In many cases, earphones must be corrected in such a way that the transfer function between the electrical input of the earphone and a pickup point in the ear canal meets a desired specification. We therefore studied the earphones on the ear of a test person. Under these conditions one finds group delay curves which seem to resemble those of comb filters. The structure of the curves is quite similar for both of the earphones tested and apparently does not depend much on the location of the pickup point in the ear canal. It has to be assumed that these all-pass characteristics stem mainly from the pinna and the ear canal and only to a small extent from the earphone itself.

Figure 3 shows the all-pass content of three typical

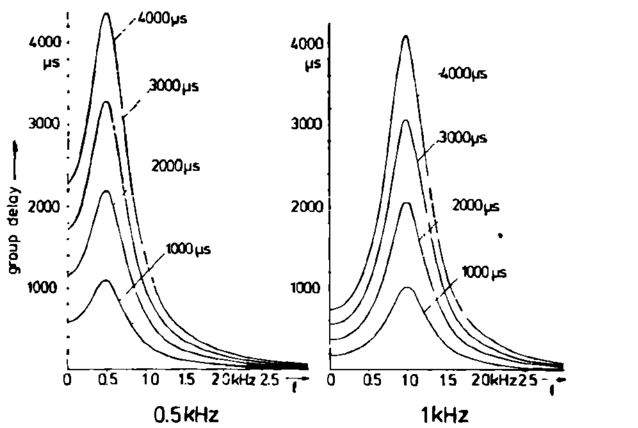


FIG. 5. Group delay patterns for preliminary hearing tests.

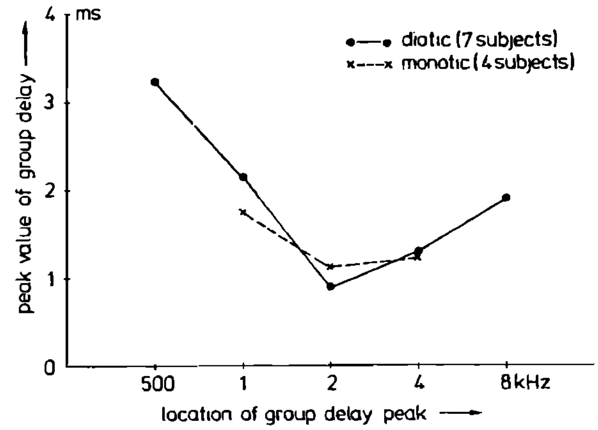


FIG. 7. Threshold of perceptibility for group delay patterns of Figs. 4 and 5. Test signal: bandlimited sound impulses.

loudspeaker systems of medium quality in typical enclosures. Type Philips 9710 M is a broadband system with a coaxial high-frequency tweeter, build into a closed 16 dm^3 box. The group delay peak near 8 kHz might be explained by a shift of the radiating plane when the high-frequency dome takes over from the main membrane.

The Isophon KSB 12/8 and Heco B 180/M loudspeakers are both two-way closed boxes. The crossover frequency is about 2 kHz for both systems. The increase of group delay at lower frequencies might also be due to a shift of the radiating plane.

In Fig. 4 the all-pass characteristics of a noncommercial bass reflex box of about 450-dm^3 volume are shown. The sound radiated by the rear side of the membrane is guided through an exponential horn to an outlet on the front plane of the enclosure. The measured group delays are more than three times as great as with the other three systems.

II. HEARING TESTS WITH GROUP DELAY DISTORTED SIGNALS

Test signals were group delay distorted in a specific way with the aid of all-pass filters. Subjects had to identify the distorted signals by comparison to nondistorted ones.

To produce group delay distortions, and all-pass filter bank was constructed which consisted of four first-order all-pass filters and 16 second-order all-pass filters, all connected in cascade. The filters were built up using active analog circuitry.

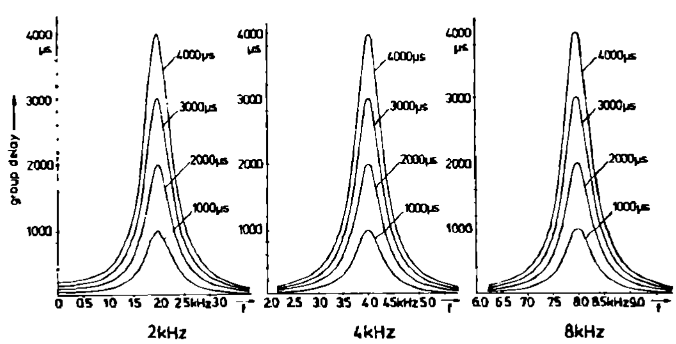


FIG. 6. Group delay patterns for preliminary hearing tests.

To compute the values of the network elements of each of the all-pass filters for the approximation of given group delay curves, a computer program was written. This program also plotted the group delay curve which is theoretically achieved when the network elements are adjusted to the computed values. A group delay measuring device after Nyquist and Brand (1930) was used to check the resulting group delay characteristic of the all-pass filter bank by direct measurement.

A. Preliminary experiments

To obtain a first estimate of the approximate limit of perceptibility of group delay distortions a preliminary hearing test with a group delay pattern as schematically shown in Figs. 5 and 6 was performed. Each distorted signal had one group delay peak either at 0.5, 1, 2, 4, or 8 kHz with a peak value of either 0, 1, 2, 3, or 4 ms.

Each distorted test signal was presented in random order together with a nondistorted signal. The subjects had to judge whether or not they heard a difference between the two signals in each pair, regardless of the attribute (absolute threshold of perceptibility). Four to seven students with normal hearing took part in these experiments. Each stimulus was presented five times to each subject.

Tests with different kinds of signals (e.g., speech, music, noise, harmonic series, and short impulses) under various conditions of presentation were carried out. It appeared that the highest sensitivity of the ear for group delay distortions can apparently be found with brief sound impulses under anechoic conditions.

In Fig. 7 we show the results for 25-µs-wide rectangular impulses, fed diotically and for some group delay peak frequencies also monotically to a set of open headphones (Sennheiser HD 414). The amplitude density spectrum of the impulses is practically constant over the frequency range between 16 Hz and 16 kHz. These impulses, however were bandlimited by a low-

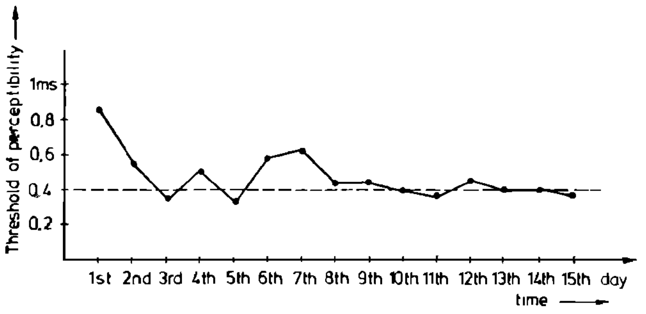


FIG. 8. Effect of training on the threshold of perceptibility for one group delay peak at 4 kHz. One subject, one training session per day. Bandlimited sound impulses.

pass filter (cutoff frequency 8 kHz, slope 24 dB/oct) and a high-pass filter (cutoff frequency 300 Hz, slope 24 dB/oct) in order to remain within the reliable frequency range of the all-pass filter bank used to produce the group delay distorted test signals. The sensation level was 40–50 SL.

The threshold values in Fig. 7 are those at which a difference between distorted and nondistorted signals was heard with a relative frequency of 50%. Differences for monotic and diotic presentation are not significant.

All subjects were untrained in these tests. However, it appeared that their performance increased during the test. For this reason we carried out the following training experiment.

B. Effect of training

The training experiment was performed with one normal-hearing subject. Test signals were again bandlimited 25-μs-wide rectangular impulses. To make judgments easier, each stimulus consisted of a series of five impulses, following each other with a 300-ms interval. The level was 50 dB SL. Presentation was diotic via a set of open headphones (HD 414). The distorted stimuli had an additional group delay peak at 4 kHz as shown in Fig. 6, but also with lower peak values (0.25, 0.50, 0.75, and 1 ms).

Further, a more sensitive psychometric method was used, i.e., a triadic method. The distorted stimulus was presented together with two nondistorted stimuli in random order. The subject had to identify the distorted stimulus within this group of three stimuli in a forced-choice procedure. The threshold is defined as that peak group delay value at which a correct identification is achieved with a relative frequency of 67%. Training sessions were performed during a period of 15 days at 9 a.m. each day and each session lasted 30 minutes. The subject received feedback after every judgment and had to repeat the judgment in case of failure. Each session began with a measurement. The results are plotted in Fig. 8.

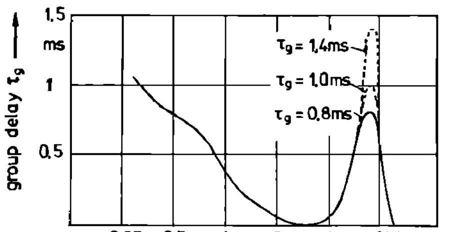
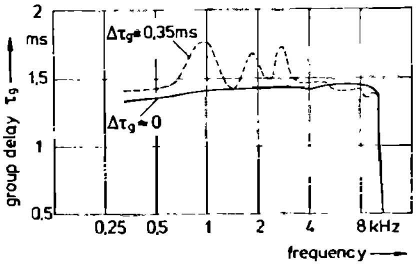


FIG. 9. Group delay pattern A of main experiments. Seven trained subjects, bandlimited sound impulses.



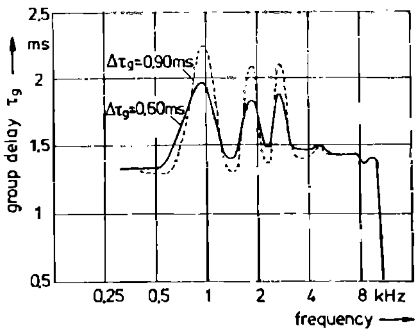


FIG. 10. Group delay pattern B of main experiments. Fifteen trained subjects, bandlimited sound impulses.

After only one day of training we found a reduction in threshold values from 0.86 to 0.54 ms, and in the following session the asymptotic value of 0.4 ms has already been reached. The threshold values stabilized after the eighth day.

We learned from these tests that even relatively short training is sufficient to make subjects familiar with the attributes of sound sensations which are correlated to group delay distortions. Subsequent tests were therefore always preceded by appropriate training sessions.

C. Main experiments

These experiments were carried out with group delay distortion patterns which are approximations to those actually measured (see Sec. I). The triadic method described in the previous section was used. Test signals were again sequences of five bandlimited 25-μs-wide rectangular impulses. The signal level was 50 dB SL.

Seven trained students took part in the tests with

TABLE I. Results of hearing tests with group delay pattern A.

Peak group delay values	0.8 ms	1.0 ms	1.4 ms
Relative frequency of correct judgments	47.6%	62.8%	75.2%
Threshold of perceptibility		$\tau_g \approx 1.1$ ms	

TABLE II. Results of hearing tests with group delay pattern B.

Group delay differences (peaks to valleys)	0 ms	0.35 ms	0.60 ms	0.90 ms
Relative frequency of correct judgments	30.9%	57.3%	77.2%	91.5%
Threshold of perceptibility	$\Delta\tau_g \approx 0.5$ ms			

group delay pattern A, 15 trained students in those with pattern B. Each stimulus was presented 24 times to each student. Presentation was diotic by a set of open earphones (Sennheiser HD 414) in a soundproof room.

Group delay pattern A is shown in Fig. 9. It is an approximation of the all-pass characteristics of the Philips 9710 M loudspeaker (see Fig. 3), whereby the peak value of the group delay peak at 7 kHz was varied. The relative frequencies of correct judgments are shown in Table I. The threshold of perceptibility for this pattern is reached when the peak value of the 7-kHz peak is about $\tau_g = 1.1$ ms.

Group delay pattern B is shown in Fig. 10. It resembles the group delay characteristics of earphones on the ear (see Fig. 2).

For this stimulus pattern the low-pass cutoff frequency was set a 6 kHz with a slope of 72 dB/oct, so that all signal components above 9 kHz were completely inaudible. The extreme slope of group delay above 9 kHz can thus be neglected. The relative frequencies of correct judgments are shown in Table II. The threshold of perceptibility for this pattern is given by a group delay difference (peaks to valleys) of about $\Delta\tau_g = 0.5$ ms.

III. CONCLUSION

Common loudspeakers and earphones are not necessarily minimum-phase systems but show additional all-pass characteristics. The additional group delays caused by these characteristics are on the order of 400 μs.

Hearing tests under extreme conditions (i.e., trained subjects, most critical test signals, diotic earphone presentation) have established that these additional group delay distortions need not be corrected in most practical cases.

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