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LOW-FREQUENCY HORN DESIGN USING THIELE/SMALL DRIVER PARAMETERS

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LOW-FREQUENCY HORN DESIGN USING THIELE/SMALL DRIVER PARAMETERS

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The design formulas for low-frequency horns which yield various physical and performance related horn data can be recast in a form which utilizes the Thiele/Small direct-radiator driver parameters. This conversion simplifies computations of items such as required back cavity volume and throat area for desired performance. Performance data such as operating bandwidth, upper rolloff frequencies and low-frequency maximum acoustic output power are easily calculated.

INTRODUCTION

For purposes of direct-radiator loudspeaker system analysis and design, it has been found advantageous to describe the driver in terms of four basic parameters used by Thiele [1] and Small [2] which are related to the fundamental electromechanical driver parameters but are easier to measure and work with. These advantages can be extended to the design and analysis of low-frequency exponential horn systems if the appropriate equations are rewritten in a form which utilizes the Thiele/Small driver parameters.

GLOSSARY OF SYMBOLS

B	magnetic flux density in driver air gap
c	velocity of sound in air (=343 m/s)
C_{AB}	acoustic compliance of air in enclosure
C_{MES}	electrical capacitance due to driver mass including rear air load (=$M_{MS}/(B^2\ell^2)$)
C_{MET}	electrical capacitance which varies with frequency due to horn throat air load mass (=$\rho_0 c\, S_D^2/(2\pi B^2\ell^2\, S_T\, f_c)$ for infinite exponential horn, valid for $f \geqslant f_c$ only)
C_{MS}	mechanical compliance of driver suspension
e_{in}	voltage applied to driver terminals
f	frequency
f_c	horn cutoff frequency
f_{HC}	upper rolloff corner (-3 dB) frequency due to the effects of front cavity compliance acting alone
f_{HM}	upper rolloff corner (-3 dB) frequency due to the effects of driver moving mass acting alone
f_{HS}	upper frequency bound of the driver's resistance controlled region when operated in free air

f_{HVC}	upper rolloff corner (-3 dB) frequency due to the effects of driver voice coil inductance acting alone
f_{LBC}	lower rolloff corner (-3 dB) frequency due to driver suspension and back cavity compliance when driving infinite tube
f_{LC}	lower rolloff corner (-3 dB) frequency due to driver suspension compliance alone when driving infinite tube
f_{LS}	lower frequency bound of the driver's resistance controlled region when operated in free-air
f_S	resonance frequency of driver in free-air
ℓ	length of voice-coil conductor in magnetic field
L_{CEB}	electrical inductance due to compliance of air in back cavity $(=B^2\ell^2V_B/(\rho_0 c^2S_D{}^2))$
L_{CEC}	electrical inductance due to compliance of air in front cavity $(=B^2\ell^2V_{FC}/(\rho_0 c^2S_D{}^2))$
L_{CES}	electrical inductance due to driver suspension compliance $(=B^2\ell^2\ C_{MS})$
L_E	inductance of driver voice-coil
M_{MS}	mechanical mass of driver diaphragm assembly including back air load
P_A	acoustic output power
P_{AR}	displacement-limited acoustic power rating
P_E	nominal electrical input power $(=e_{in}{}^2/(2R_E)\ )$
Q	ratio of reactance to resistance (series circuit) or resistance to reactance (parallel circuit)
Q_{ES}	Q of driver at f_S considering electrical resistance R_E only
Q_{MS}	Q of driver at f_S considering mechanical losses only
Q_{TS}	total Q of driver at f_S including all system resistances $(=Q_{MS}\ Q_{ES}/(Q_{MS}+Q_{ES})\ )$
R_E	dc resistance of driver voice coil
R_{ET}	electrical resistance which varies with frequency due to power radiated into horn (proportional to horn throat conductance)
S_D	effective projected surface area of driver diaphragm
S_T	throat area of horn
V_B	net internal volume of rear cavity $(\approx\rho_0 c^2\ C_{AB})$
V_D	peak displacement volume of driver diaphragm $(=S_D\ x_{MAX})$

V_{AS}	volume of air having some acoustic compliance as driver suspension (=$\rho_0 c^2 C_{MS} S_D^2$)
V_{FC}	net internal volume of front cavity
x_p	peak displacement of driver diaphragm
x_{MAX}	maximum peak linear displacement of driver diaphragm
α	compliance ratio between driver suspension compliance and compliance of air in rear cavity (also=V_{AS}/V_B)
β	compliance ratio between driver suspension compliance and compliance of air in front cavity (also=V_{AS}/V_{FC})
η	efficiency
η_o	reference efficiency (=acoustic output power/nominal electrical input power)
ρ_0	density of (=1.21 kg/m3) air

REVIEW

Driver Parameters

The fundamental electromechanical driver parameters which control system low-frequency performance are [2, p. 387] R_E, (Bl), S_D, C_{MS}, M_{MS}, R_{MS}, and x_{MAX} which are defined in the glossary of symbols. These parameters are directly related to the drivers' physical characteristics such as diaphragm suspension compliance, total moving mass, the strength of the magnetic field, etc.

Another set of driver descriptors which are related to those above have been gaining increased usage because they are easier to measure and simplify the system design process. These are the parameters f_S, V_{AS}, Q_{TS} (=$Q_{ES} Q_{MS}/(Q_{ES}+Q_{MS})$) and V_D used by Thiele [1] and Small [2] and defined in the symbol glossary. These parameters are more closely associated with directly measurably quantities such as resonance frequency and Q. The conversion between these two sets of parameters is outlined in Appendix .

Low Frequency Horn Design:

Traditional low frequency exponential horn design and analysis using cone type drivers deals with such items as [3]; [4], [5], [6], [7], [8], [9], [10]:

1. Selection of horn cutoff frequency and flare rate for desired performance.
2. Selection of throat area to maximize efficiency.
3. Selection of mouth area for best response.
4. Selection of back cavity volume for reactance annulling at horn cutoff.
5. Computation of low-frequency maximum acoustic output power.
6. Computation of high frequency rolloff corner frequencies due to driver moving mass, driver voice coil inductance and front cavity compliance.

This paper will deal only with Items 2, 4, 5, and 6 with emphasis on designs where a horn must be designed for a given driver. For Item 5, only displacement limited maximum output will be analyzed.

Horn Equivalent Circuit

The simplified electrical equivalent circuit of the horn-driver system of Fig. 1 is shown in Fig. 2 [3, p. 262]. Symbols correspond to that used by Small [2]. Driver and box resistive losses are neglected.

Efficiency

The method used in this paper to compute efficiency is similar to that used by Beranek [3, p. 262] and Small [2] and is defined as the acoustic output power divided by the nominal electrical input power delivered by the source into a resistor having a value twice the rated DC voice-coil resistance $(P_{in}=e_{in}^2/(2R_E))$.

For midband operation, the efficiency is maximized at a value of 50% when the reflected load resistance equals the driver's voice coil resistance ie $R_{ET} \approx R_E$. This situation can be attained for a specific throat area given by [6, p. 279], [10, eq. 3 if n=1]:

$$S_T = \frac{\rho c\, R_E\, S_D^2}{B^2 \ell^2} \qquad (1)$$

It must be noted that the widest bandwidth may not be obtained for this maximum efficiency situation.

Frequency Response

As Beranek indicates [3, pp. 263-266], the frequency response of a horn system can be divided into three distinct regions: low, mid, and high frequencies. If the throat impedance of the horn is assumed to be purely resistive and constant with frequency (simulates a horn with very low cutoff or infinite tube load) the response or nominal efficiency versus frequency can be modeled as shown in Fig. 3. The three frequency bands along with indicated corner points are clearly shown. The three regions indicate respectively compliance, resistance and mass controlled portions of horn operation.

As an aid to later analysis, it helps to define two driver related corner frequencies which indicate respectively the approximate upper and lower bounds of the resistance controlled region of the unmounted driver:

Upper bound,

$$f_{HS} = \frac{B^2 \ell^2}{2\pi R_E M_{MS}} \quad ; \text{ and} \qquad (2)$$

Lower bound,

$$f_{LS} = \frac{R_E}{2\pi B^2 \ell^2 C_{MS}} \quad . \qquad (3)$$

Note that $f_S = \sqrt{f_{LS} f_{HS}}$.

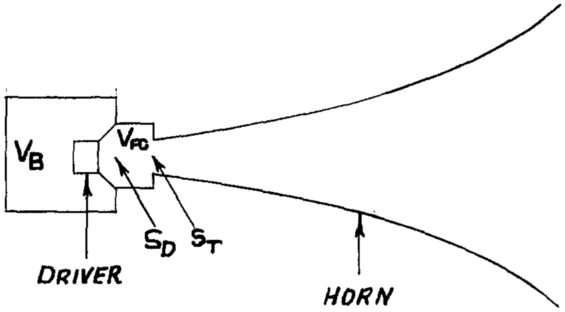

Fig. 1. Depiction of low-frequency horn-driver system. Back cavity V_B, front cavity V_{FC}, diaphragm area S_D, and horn throat area S_T are indicated.

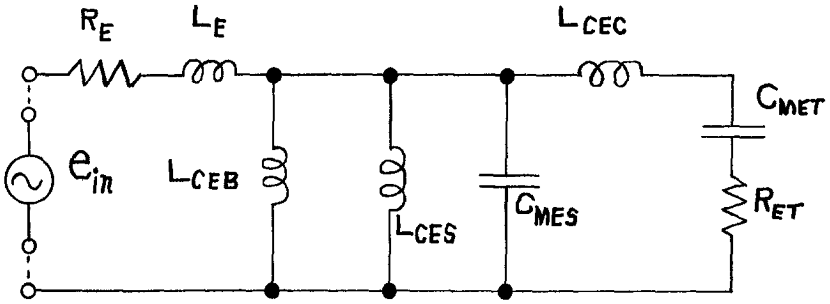

Fig. 2. Simplified lumped electrical equivalent circuit of the low-frequency horn-driver system depicted in Fig. 1. Symbols are defined in glossary of symbols. The effects of driver mechanical resistive losses have been neglected ($Q_{MS} \gg Q_{ES}$). The horn's throat load appears as R_{ET} and C_{MET} which are both non-constant functions of frequency in the general case.

Low Frequencies

At low frequencies, the simplified electrical equivalent circuit reduces to the form shown in Fig. 4a. Examination reveals that the response rolls off at 6 dB per octave below a frequency set by certain driver parameters including suspension compliance, effective circuit resistance, and back cavity compliance.

If the efficiency is maximized by setting the throat area to the value in eq. (1), and the effects of back cavity compliance are neglected $(V_B \rightarrow \infty)$, the lower driver compliance corner frequency is given by:

$$f_{LC} = \frac{R_E}{4\pi B^2 \ell^2 C_{MS}} = f_{LS}/2. \qquad (4)$$

For a finite back cavity, the lower corner frequency is increased to:

$$f_{LBC} = \frac{R_E\,(1+\alpha)}{4\pi B^2 \ell^2 C_{MS}} = f_{LC}\,(1+\alpha) = \frac{f_{LS}\,(1+\alpha)}{2} \qquad (5)$$

Where $\alpha = C_{MS}/C_{AB}$, the ratio between the driver suspension compliance and the box compliance.

Mid Frequencies:

At mid frequencies the equivalent circuit reduces to Fig. 4b. Analysis yields a maximum midband nominal efficiency of

$$\eta = \frac{2\,R_E\,R_{ET}}{(R_E+R_{ET})^2} \qquad (6)$$

where $R_{ET} = S_T\,B^2\ell^2/(\rho\,c\,S_D^2)$,

which is maximized when $R_{ET}=R_E$ by setting S_T according to eq. (1).

High Frequencies:

At high frequencies the equivalent circuit takes the form shown in Fig. 4c which is a 3rd-order low-pass filter. Three individual rolloff mechanisms are exhibited which are

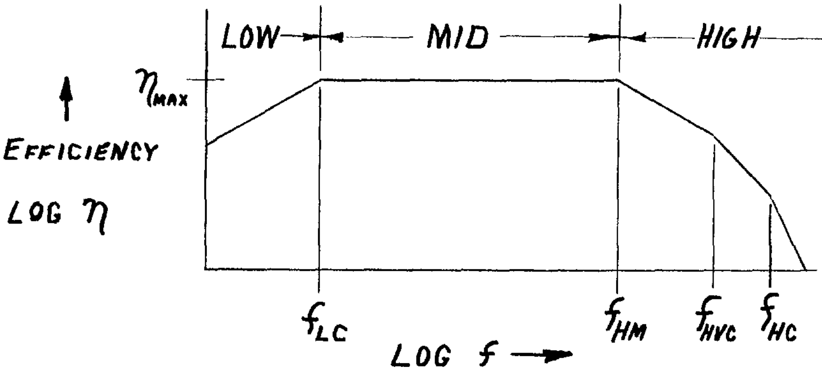

Fig. 3. Idealized frequency response of horn-driver system. Horn cutoff frequency f_c is assumed to be very much lower than f_{LC} (C_{MET} very large). The midrange band is defined primarily by driver and back cavity compliance rolloff on the low end ($f<f_{LC}$) and driver effective moving mass rolloff on the high end ($f>f_{HM}$). Secondary high-end rolloffs due to driver voice-coil inductance ($f>f_{HV}$) and front cavity compliance ($f>f_{HC}$) are exhibited.

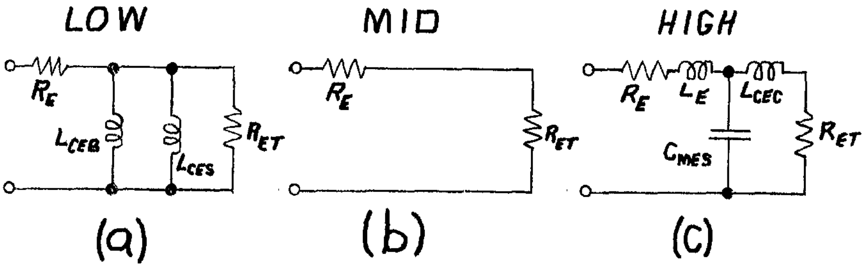

Fig. 4. Reductions of the horn-driver system simplified electrical equivalent circuit of Fig. 2 in each frequency band indicated in Fig. 3. It is assumed that $f_C \ll f_{LC}$ as in Fig. 3.

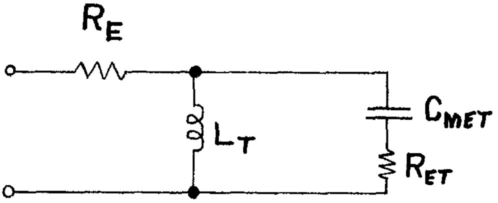

Fig. 5. Reduction of the simplified horn-driver system electrical equivalent circuit of Fig. 2 in the low frequency band but considering the effects of horn cutoff. Note that in this region both C_{MET} and R_{ET} are non-constant functions of frequency. For the case of an infinite exponential horn however, C_{MET} is constant and positive above cutoff ($f \geq f_C$). Note that $L_T = L_{CEB} L_{CES}/(L_{CEB}+L_{CES})$.

dependent on driver moving mass, driver voice coil inductance, and front air-chamber compliance.

If the relationship of eq. (1) holds, analysis reveals individual breakpoint frequencies of:

1. Driver moving mass corner,

$$f_{HM} = \frac{B^2 \ell^2}{\pi R_E M_{MD}} \approx 2 f_{HS}; \qquad (7)$$

2. Driver voice coil inductance corner,

$$f_{HVC} = \frac{R_E}{\pi L_E} \quad ; \quad \text{and} \qquad (8)$$

3. Front cavity compliance corner,

$$f_{HC} = \frac{2 \rho_0 c^2 R_E S_D^2}{B^2 \ell^2 V_{FC}} \qquad (9)$$

where V_{FC} is the volume of the front cavity.

In a real world horn design, the composite high frequency rolloff is a complex combination of all three corner frequencies taken together. These three frequencies do give a designer a rough idea of the high frequency behavior of the system, however. In a practical situation these breakpoints are often ordered as $f_{HM} < f_{HVC} < f_{HC}$.

Reactance Annulling

The low frequency efficiency of a horn loaded system at frequencies near horn cutoff may be increased somewhat by minimizing the effects of the horn's throat air mass reactance by a process known as reactance annulling. This method, which was first used by Klipsch [5] and later refined by Plach and Williams [6] [7], uses the compliance reactance of the combined effects of the driver's suspension and rear cavity compliances to offset the horn's throat mass reactance.

Analysis of the equivalent circuit at low frequencies, with the appropriate throat resistive and reactive values substituted for an infinite exponential horn [9, eq. 4.7] (shown in Fig. 5), reveals that reactance annulling is the same as equating the lower bound of the resistance controlled region of the driver mounted in its closed-box rear cavity to the horn's cutoff frequency:

$$f_{LS}\,(1+\alpha) = f_C = 2\,f_{LBC}\ . \qquad (10)$$

With the information that $\alpha = C_{MS}/C_{AB}$ and

$$C_{AB} = \frac{V_B}{\rho_0\, c^2\, S_D^2}\ . \qquad (11)$$

where V_B is the effective rear cavity volume, eq. (10) may be solved for V_B yielding:

$$V_B = \frac{\rho_0\, c^2\, S_D^2\, C_{MS}}{\dfrac{2\pi f_C\, B^2 \ell^2\, C_{MS}}{R_E} - 1}\ . \qquad (12)$$

If the total compliance is set primarily by the box ie $C_{AB} \ll C_{MS}$. eq (12) reduces to:

$$V_B = \frac{\rho_0\, c^2\, R_E\, S_D^2}{2\pi f_C\, B^2 \ell^2}\ . \qquad (13)$$

Eqs. (13) and (1) may be combined to yield:

$$V_B = \frac{S_T\, c}{2\pi f_C} = \frac{S_T \lambda C}{2\pi} \qquad (14)$$

where λ_C = wavelength at cutoff, which is a simple practical form first derived by Klipsch in 1941 [5, eq. 3].

Low-Frequency Maximum Acoustic Output

The maximum acoustic output of the horn system at low frequencies is primarily set by the maximum displacement capabilities of the driver, the maximum thermal capabilities of the driver, and non-linear air compression distortion in the back cavity.

Considering only the driver's displacement limitations, the power radiated into an infinite tube of area S_T by a flat piston of area S_D undergoing sinusoidal oscillations of peak amplitude x_p is given by Olson [4, eq. 7.23]:

$$P = \frac{2\pi^2 \rho_0 c \, S_D^2 \, x_p^2 \, f^2}{S_T} \quad . \qquad (15)$$

This expression can be rewritten in terms of the horn's cutoff frequency f_C and the maximum low-frequency displacement limited output power P_{AR}, by noting that for a well designed finite exponential horn with optimum mouth size [8], the low-frequency efficiency is down no more than 0.3 dB from the maximum midband efficiency at 1.26 f_C.

Therefore:

$$P_{AR} \approx \frac{3\pi^2 \rho_0 c \, S_D^2 \, x_p^2 \, f_C^2}{S_T} \quad . \qquad (16)$$

This equation may be combined with eq. (1) to yield:

$$P_{AR} \approx \frac{3\pi^2 \, B^2 l^2 \, x_p^2 \, f_C^2}{R_E} \quad . \qquad (17)$$

CONVERSION

The relationships noted in Appendix I can be used to rewrite eqs. (1)-(5), (7)-(9), (12), and (16) in terms of the Thiele/Small driver parameters. In all cases $Q_{TS} \approx Q_{ES}$, due to the assumption that $Q_{MS} \gg Q_{ES}$.

Efficiency

The expression giving the midband nominal efficiency eq. (6) remains the same, but the value of horn throat area to maximize this function eq.(1) may be written as:

$$S_T = \frac{2\pi f_S \, Q_{TS} \, V_{AS}}{c} \quad , \tag{18}$$

which is the desired result.

Frequency Response

The driver related corner frequencies which indicate the bounds of resistance controlled operation can be shows as:

$$f_{HS} = f_S / Q_{TS}, \text{ and} \tag{19}$$

$$f_{LS} = Q_{TS} \, f_S \quad . \tag{20}$$

These bounds roughly indicate the range over which a driver will be suitable for use as a horn driver considering small-signal operation only.

It is instructive to form the ratio of these two expressions ie $f_{HS}/f_{LS}=1/Q_{TS}^2$, which indicates that a low value of Q_{TS} (high motor strength, large magnets, etc.)is desirable for loudspeakers used as horn drivers if the widest operating bandwidth is desired.

Low Frequencies

Eqs. (4) and (5) can be rewritten as:

$$f_{LC} = f_{LS}/2 = Q_{TS} \, f_S/2, \text{ and} \tag{21}$$

$$\begin{aligned} f_{LBC} = f_{LC} \, (1+\alpha) &= \frac{Q_{TS} \, f_S}{2} \, (1+\alpha) \\ &= \frac{Q_{TS} \, f_S}{2} \left(1 + \frac{V_{AS}}{V_B}\right) , \end{aligned} \tag{22}$$

where $\alpha = V_{AS}/V_B$ the ratio between the driver's compliance equivalent volume and the rear cavity box volume.

Mid Frequencies

The efficiency expression eq. (6) remains the same as noted before.

High Frequencies

The three HF breakpoint frequencies eqs. (7)-(9) can be shown in the form:

1. Driver moving mass corner,

$$f_{HM} = 2f_{HS} = 2f_S/Q_{TS}; \tag{23}$$

2. Driver voice coil inductance corner remains as before eq. (8); and
3. Front cavity compliance corner,

$$\begin{aligned} f_{HC} &= 2f_{LS}\beta = 2Q_{TS}\ f_S\ \beta \\ &= 2Q_{TS}\ f_S\ \frac{V_{AS}}{V_{FC}} \end{aligned} \tag{24}$$

where $\beta = V_{AS}/V_{FC}$ the ratio between the driver's compliance equivalent volume and the front cavity volume.

Reactance Annulling

The correct rear cavity volume for reactance annulling eq. (12) can be changed to:

$$V_B = \frac{V_{AS}}{\left(\frac{f_C}{f_{LS}} - 1\right)} = \frac{V_{AS}}{\left(\frac{f_C}{f_S\ Q_{TS}} - 1\right)}, \tag{25}$$

which is a relatively direct compact form. It must be noted that normally $(f_C/f_{LS}) \angle 1$ or $f_{LS} < f_C$ which makes V_B finite and positive. If $f_{LS} \approx f_C$ or $f_{LS} > f_C$, the driver is not well suited for operation in a horn at that specific cutoff frequency.

Low-Frequency Maximum Acoustic Output:

The expression for the displacement limited low-frequency output power eq. (16) can be combined with eq. (18) yielding:

$$P_{AR} = \left(\frac{3}{2}\pi\rho\ c^2\right)\left(\frac{1}{f_S\ Q_{TS}\ V_{AS}}\right)\ f_C^2\ V_D^2\ . \tag{26}$$

For computation in SI metric units $3\pi\rho c^2/2 \approx 6.7 \times 10^5$.

COMPARISON

A comparative listing of some of the horn design equations considered and developed in this paper are shown in Table I.

TABLE I.

A comparison of horn design equations between those which use the fundamental electromechanical driver parameters and the Thiele/Small driver parameters.

Symbol	Description	Electromechanical	Thiele/Small
S_T	Horn throat area	$\frac{\rho_0 \, c \, R_E \, S_D^2}{B^2 \ell^2}$	$\frac{2\pi f_S \, Q_{TS} \, V_{AS}}{c}$
V_B	Back cavity volume	$\frac{\rho_0 \, c^2 \, R_E \, S_D^2 \, C_{MS}}{2\pi f_c \, B^2 \ell^2 \, C_{MS} - R_E}$	$V_{AS} \Big/ \left(\frac{f_C}{f_S Q_{TS}} - 1 \right)$
	if $V_{AS} \gg V_B$	$\frac{S_T \, c}{2\pi f_C}$	$V_{AS} \, f_S \, Q_{TS}/f_C$
HF rolloff corner frequencies			
f_{HM}	Due to moving mass	$\frac{B^2 \ell^2}{\pi R_E \, M_{MD}}$	$\frac{2 \, f_S}{Q_{TS}}$
f_{HVC}	Due to voice coil inductance	$\frac{R_E}{\pi L_E}$	Same
f_{HC}	Due to front cavity	$\frac{2 \rho_0 \, c^2 \, R_E \, S_D^2}{B^2 \ell^2 \, V_{FC}}$	$2 \, Q_{TS} \, f_S \left(\frac{V_{AS}}{V_{FC}} \right)$
P_{AR}	Displacement limited max. acoustic output	$\frac{3\pi^2 \, B^2 \ell^2 \, X_p^2 \, f_C^2}{R_E}$	$\frac{3\pi \rho_0 \, c^2}{2 f_S \, Q_{TS} \, V_{AS}} \quad f_C^2 \, V_D^2$

DESIGN EXAMPLE

A low-frequency exponential horn system with cutoff f_C=50 Hz is to be designed for a typical high-efficiency musical instrument driver. Details of horn flaring and selection of proper mouth size will not be considered here but are covered in [3], [4], [8], [9].

Driver Parameters:

The parameters of the 12 inch driver to be used in the horn are listed as follows (all free-air, unenclosed):

Electromechanical Parameters:

M_{MS} = 31.4 g (includes air mass load)
C_{MS} = 4.0×10^{-4} m/N
B = 15.2 Tm
R_E = 5.6 Ω
Mechanical Q = 9.5
x_{max} = 3.3 mm
S_D = 5.0×10^{-2} m^2
L_E = 3.2 mH

Thiele/Small Parameters:

f_S = 45 Hz
Q_{ES} = 0.215
Q_{MS} = 9.5
Q_{TS} = 0.210
V_{AS} = $140\ell = 0.14\ m^3$
η_0 = 5.8% (half-space)
V_D = $0.166\ell = 1.66 \times 10^{-4}\ m^3$
P_E (max) = 100 Watts

Design:

Application of eq. (18) yields for throat area

$$S_T = \frac{2\pi(45)\ (0.21)\ (0.14)}{343} = 2.4 \times 10^{-2}\ m^2$$

$$= 242\ cm^2,$$ and eq. (25)

for back cavity volume

$$V_B = \frac{140}{\frac{50}{45(0.21)} - 1} = 32.6\ \ell$$

$$= 3.26 \times 10^{-2}\ m^3.$$

Analysis:

Small Signal:

The upper and lower bounds of the driver's resistance controlled region are given by eqs. (19 and (20):

$$f_{HS} = f_S/Q_{TS} = 45/0.21 \approx 214\ Hz$$ and

$$f_{LS} = Q_{TS}\ f_S = 0.21\ (45) \approx 9.5\ Hz.$$

High Frequencies:

The three HF rolloff breakpoints from eqs. (23), (8), and (24) are:

1. Driver moving mass corner,

$$f_{HM} = 2\ f_{HS} \approx 430\ Hz;$$

2. Driver voice coil inductance corner,

$$f_{HVC} = R_E/(\pi L_E) = 5.6/(\pi .0032) \approx 560 \text{ Hz; and}$$

3. Front cavity compliance corner ($V_F = 1.1\ \ell$),

$$f_{HC} = 2 f_{LS} \frac{V_{AS}}{V_F} = \frac{2\ (9.5)\ (140)}{1.1} \approx 2400 \text{ Hz.}$$

These breakpoints indicate a 6 dB/octave rolloff starting at 430 Hz, 12 dB/octave at 560 Hz, and a 18 dB/octave rolloff above 2,400 Hz.

Reactance Annulling:

To check for proper reactance annulling the relationship of eq. (10) can be checked:

$$f_{LS}\ (1+\alpha) = f_{LS}\left(1 + \frac{V_{AS}}{V_B}\right) = 9.5\ \left(1 + \frac{140}{32.6}\right)$$

$$\approx 50 \text{ Hz,}$$

which is equal to the cutoff frequency as desired.

Large Signal:

The displacement limited LF acoustic output power from eq. (26) is:

$$P_{AR} = \frac{6.7 \times 10^5\ (50)^2\ (1.66 \times 10^{-4})^2}{45\ (0.21)\ (0.14)}$$

$$\approx 35 \text{ Watts.}$$

This indicates that the system is capable of generating some 35 acoustic watts or more down to 1.26 $f_C \approx 63$ Hz without exceeding the driver's rated maximum displacement of ± 3.3 mm (± 1/8th inch). The other limiting mechanism of low-frequency output is the driver's maximum thermal power rating P_{AR}, which is not considered in this analysis.

CONCLUSION

For those who prefer design methods using the Thiele/Small driver parameters, this paper has developed a set of equations for low-frequency horn design which use these parameters. If the Thiele/Small parameters are known for a particular driver, the horn system may be designed and analyzed using these rewritten equations. In some cases, simplifications in design and analysis result from these transformed equations.

It must be pointed out that the transformed design formulas used in this paper are based on traditional low-frequency horn design methods. These traditional methods under some situations may not yield a design which has the optimum combination of response, efficiency and maximum acoustic output. This is primarily due to the fact that traditional horn design dictates a specific value of throat area which maximizes the nominal efficiency. Because a number of the horn's performance characteristics depend heavily on throat area, constraint of this parameter to a specific value removes one valuable degree of design freedom.*

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* These last comments resulted from private correspondence with Dr. Richard H. Small of the University of Sydney, Australia.

APPENDIX

CONVERSION BETWEEN ELECTROMECHANICAL DRIVER PARAMETERS AND THIELE/SMALL DRIVER PARAMETERS

The Thiele/Small driver parameters are related to the electromechanical driver parameters by the following relationships [2]:

$$f_S = \frac{1}{2\pi} \sqrt{\frac{1}{M_{MS}\, C_{MS}}} \quad , \qquad (27)$$

$$Q_{ES} = \frac{R_E}{B^2 \ell^2} \sqrt{\frac{M_{MS}}{C_{MS}}} \quad , \qquad (28)$$

$$Q_{MS} = \frac{1}{2\pi\, f_S\, C_{MS}\, R_{MS}} \quad , \qquad (29)$$

$$Q_{TS} = \frac{Q_{MS}\, Q_{ES}}{Q_{MS} + Q_{ES}}$$

if $Q_{MS} \gg Q_{ES}$ then $Q_{TS} \simeq Q_{ES}$,

$$V_{AS} = \rho_0\, c\, S_D^{\,2}\, C_{MS}, \text{ and} \qquad (30)$$

$$V_D = S_D\, x_{max}. \qquad (31)$$

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