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Frequency Domain Experiences in Loudspeaker's Suspensions

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ABSTRACT

The article proposes to analyze individually each part that makes the loudspeaker up, specifically the diaphragm-surround set. Experiences were performed on low and medium amplitude displacement ranges. The paper uses traditional experimental methods to seek for the surround and diaphragm's spectral signatures in the main eigenvalue region. Method consists in exciting the diaphragm-surround set by a reluctance transducer which was fed by an electric impulse, and analyze its response with an Eddy Current Displacement Transducer in the Frequency Domain. The most typical experimental spectral signatures of the nonlinear systems in free response are reviewed. This paper presents the results that were obtained after examining six samples, finding only one sample completely free of nonlinearities.

1. HISTORICAL BACKGROUND AND AIMS

As the reader knows, it is experience what validates a theory. But when we talk about acoustic transducers for sound reproduction, it is generally subjective evaluation what finally validates it. It is a fact that, in electroacoustics, complex subjective behaviors are justified with few and even insufficient measurements. The measurement of vectorial frequency response function, the measurements of some kinds of distortion, and the impulse response are not enough in

some cases to justify the subjective acoustic of a loudspeaker. This is the reason why the technical and scientific communities are proposing new measurements, but many of them do not seem to consolidate and end up by not being applied in the manufacturer's development laboratories.

This work offers some measurements and some evaluation criteria that are complementary to those already mentioned above. The paper sets out to use experimental methods besides calculus. Calculus methods have spread widely and have been divulged far more than experimental methods. On the other hand, as transmitting transducers are a part of the world of dynamic systems, one of their most important aspects is their behavior in its eigenvalues region, particularly for the lower ones. For this reason, the paper concentrates in this aspect.

In the nonlinear domain, a domain that it is difficult to escape from, we need to tend, as it is natural, to modelize it and treat it analytically. Meanwhile, we need to give development engineers criteria so that they can design high performance products. These criteria have some parts that can be expressed in an analytical form, and other parts that cannot be expressed as such yet.

The bibliography at the end of the paper illustrates many efforts to describe and analyze with mathematical models the behavior in several nonlinear systems.

Summing up, the paper aims to enlarge the criteria that development engineers need to carry their task out, that is, to create good prototypes in a reasonable amount of time. This paper only contemplates surround and diaphragm contributions that have been paired beforehand. In this research we only paid attention to the region of the transducer's main resonance frequency. Other points of view were not contemplated.

Finally, the proposed technique is based on spectral analysis, being the most widely used technique nowadays in dynamic's laboratories.

1.1. Summarized Bibliographical Review

In 1962, professor H. Olson [1], already gave details on nonlinearities in a loudspeaker's surround. Nineteen years later, M.R. Gander [2], wrote a very detailed article on the causes of nonlinearity in complete loudspeakers, including many experiences and presenting several distortion measurement results. A year before this, F.M. Murray and H.M. Durbing [3] did an experimental study on Compression Drivers' surrounds, which is also very interesting due to its experimental essence.

In 1986 A. Dobrucki and C. Szmal [4] presented a research where they already work specifically in the transducer's resonance region. Dobrucki [5] himself extended these criteria in an article published in the Journal of the Audio Engineering Society. In 1987, J.M. Kaizer [6], wrote a very complete article dedicated to large strokes in complete loudspeakers, and in which he broke down the nonlinearities due to B1(x) and of K(x). It is a theoretical and practical paper. A year later, F. Toole's and S. Olive's article [7] was published. This article is very enlightening when it comes to the relationship between the behavior in resonance and the subjective evaluation. As we mentioned above, we cannot forget that experiences must match and justify the sensations we perceive.

W. Klippel [8] has written several articles related to nonlinear behavior in complete loudspeakers. Here we only make reference to two of them, the 1990 and the 1992 ones. This author is very active and has written extensively about this nonlinear topic. D.R. Birt [9] presented in 1991 several measurements in complete units, intensifying the study regarding B1(x). B. Zóltogórski's work [10] deals with the boundary condition effect, which surround presents for waves traveling in the diaphragm, topic we will not deal with in this article, but which is still of interest.

In 1993 M.H. Knudsen and J.G. Jensen [11] published a research that takes into account surround and spider creep and throws light on the matter. S. Temme [12] from Brüel & Kjaer Co., published a very complete Application Note, where he related nonlinearity with the distortion of complete units and also measured on several types of transducers. J.Scott, J. Kelly and G. Leembuggen [13], published in 1996 measurements taken in complete units, their graphics of Z(f) are very enlightening. The work has an important experimental charge. I. Aldoshina and others [14] show a study, mainly theoretical, of surround as an element of parametric oscillation. Parametric oscillation occurs if some of the system's constants (such as the spider's or the surround's stiffness) are not constant but periodical.

In 1999 K. Satoh and others [15] published experimental units tests that are also complete. In their experiences they show surround's hysteresis. In the year 2000 A. Bright [16] presented a research in which he showed the suspended diaphragms modes where there is rotation or rocking. We shall see that this circumstance is important in the development of a speaker's surround. In this same year, S. Hutt [17] showed in his article many details of the nonlinear spiders. This paper is highly recommended for those interested in the nonlinearity of the surround and spider elements. The author shows experimental data, in which hysteresis is included.

Another worth mentioning research is S.T. Park's and S.Y. Hong's article [18] from the year 2000 were the authors break down the nonlinear topic for loudspeakers. They apply rather important voltages to complete units and analyze the nonlinear behavior in the diaphragm when long strokes occur, especially from the distortion's point of view. In 2002 D.B. Keele Jr. and

R.J. Milhelich [19] wrote a theoretical-practical paper explaining the bouncing that takes place when the diaphragm moves in a complete unit for very large strokes. The bouncing study the author proposes offers very good perspectives when testing the unit's maximum strokes.

1.2. The Individual Study of the Moving Parts

In the development phase, it is obvious that the optimum transducer is made up by optimum subsets. In this way, the optimum transducer for an application has an optimum motor, optimum moving parts, an optimum frame, etc. The optimum moving parts, in turn, as they are suspended by the surround and by the spider, require elements which have been individually optimized in a specific way.

This is why the study of the set formed by the surround and the mass it is associated to, the diaphragm, has a very important and specific weight in the behavior of the transducer. The more we know about this surround, the better we shall design the complete transducer. This would be so because we would then be able to choose for the most adequate surround for each need.

It should be clear that in this text the word "diaphragm" is only understood as mass, or mass distribution, which was paired to the surround as we mentioned previously. The diaphragm is also one of the suspension's two boundary conditions. The other boundary condition, the frame, should not be underrated in any way. The used frames should be infinite. This means that there must be free of eigenmodes in the frequency range of interest. There are many important qualities of the diaphragm which have a clear influence in the final result, but which are not the aim of this paper.

When we analyze the transducer we find that at times we encounter nonlinear behaviors in its main natural frequency region. The most significant case is one that most of us have seen, when we find a maximum peak in an impedance graphic that is not very symmetric or smooth. The analysis of these behaviors are the main aim of this paper. At this point it is advisable, and this is what this study proposes, to carefully analyze the spectral region in which this natural frequency is located and also its nearby range.

If we have decided to analyze the parts that make the whole transducer up separately, and to analyze carefully the first natural frequency's region, why don't we use the traditional methods and those which are typically used with suspended systems? It seems reasonable to observe the suspended diaphragm's behavior, exciting it as an isolated system and using traditional methods to do so. Another important aspect is to observe the large strokes in the units [19], but we must take into account that, when the loudspeaker reproduces sound, it spends more time working in short strokes than it does in very long ones.

The reader will remember that the Probability Density Function of a random signal has a Gauss bell shape, in which the maximum is in nil displacement. For musical and voice passages the situation is similar, the Probability's Density Function is also at its maximum in the resting region of the unit. Therefore it appears advisable to study short strokes in depth. Nevertheless, it should be mentioned that if the unit receives large current intensities, it would obviously manifest its nonlinear curve between stress and strain.

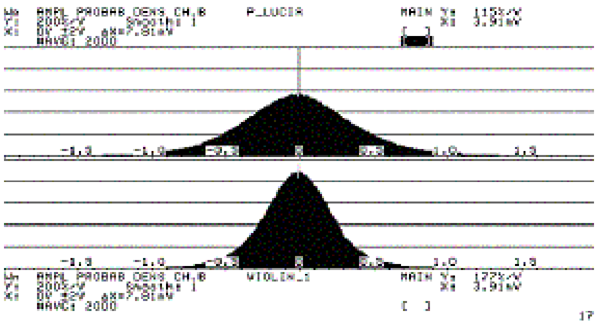

Figure 1: Amplitude Probability Density Function for two musical signals, showing the maximum values at zero amplitude. Upper figure is a Spanish guitar energetically played. Lower Figure is classical music, violin solo, gently played. Analyzer was set to the fastest sampling

Figure 1 shows this distribution for two selected musical signals. In the upper part we find the processing of a Spanish guitar passage that was played energetically (P. De Lucia, Almoraima, Bulerías). The lower part of the figure corresponds to a passage of classical music, a very gently played violin solo (Vivaldi's *Spring* in E major).

As suspensions are generally of a viscoelastic nature, or are impregnated of viscose materials, in short displacements they usually manifest a specific nonlinear characteristic. A well-known consequence is creep. Creep is the phenomenon that occurs when the displacement of the viscoelastic material increases during time and tension stays constant. References [20], [21] and [22] are all very good treatises on the subject.

This creep can be important as seen in Figure 2, where we see how two suspended diaphragms move to their resting position in a free motion after releasing them from a forced displacement. The tests of Figure 2 are done on two units to which we have removed the spiders and we have taken the measurements from the center of the dust cup. The measuring procedure of the diaphragm's movement is shown further on through an Eddy Current Transducer.

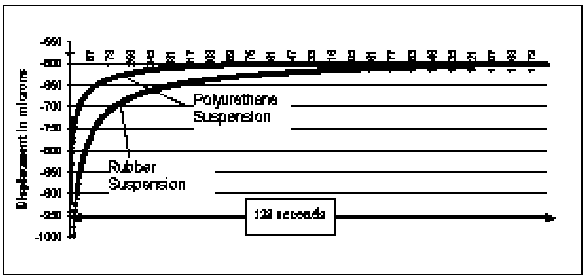

Figure 2: Displacement as a time function. Sudden release of two suspension – diaphragms, from 1mm from the measurement transducer to the rest position. Notice the slowness of the Rubber suspension sample going to the rest

The diaphragm has been displaced from its static balanced position by means of direct current applied to the coil, then we suddenly withdraw the current allowing the diaphragm to go freely and with no oscillations to its balanced position at 0 volts. This figure depicts the diaphragm's displacement as a function of time in a 128 seconds range. In it, we see how the surround made of polyurethane goes much quicker when searching for the final position than that made of rubber. The slowness of the rubber suspension is on one hand obvious, and on the other hand, rather worrying. As the reader can imagine this creep has a spectral repercussion, which we shall see later on.

This diaphragm's slow motion due to surround is not the only nonlinear behavior in a suspension. Another well known behavior of these materials is relaxation. A material's relaxation occurs when we keep a constant displacement over a material and tension gradually decreases with time.

Another very important nonlinear behavior of the viscoelastic elements is hysteresis. It is probably much better known than creep or relaxation even though it is an effect related to the same cause that originates it. It is so well known because when we apply a cyclic load (even if it is a very long period), a delay is established in the response. This delay is related with the mechanic losses. Nonlinearity caused by hysteresis is called by some authors quasistatic nonlinearity. It is so because this nonlinearity is the result of applying static tension and measuring static deformation. This nonlinearity cause is very important in any kind of suspension: in machinery shaft couplers, in vibration isolation, etc. As well as other fields such as structural analysis, seismicity, material sciences, etc. It appears in many engineering and scientific fields.

We shall not go into many descriptive details in order not to lengthen this paper too much, but in any case the loudspeakers manufacturer and the expert on dynamics and transducers know that this nonlinearity cause is almost ubiquitous. In the references we find many explanations of creep, hysteresis and experimental measurements, but [23], despite the fact that the authors work on the field of geology, is highly recommended when talking about creep and hysteresis. These authors refer to slow dynamics, a nonlinearity that could be caused by creep. Its origins are not very well known yet, although its effects are so.

Professor D. Hartog [24], pointed out that, already in the year 1920 some losses or dampings were attributed to this cause in the engine's shafts. This was so because, even though they could be in resonance, some shafts did not break due to fatigue. We should remember that creep and hysteresis are not exclusive to viscoelastic materials, they exist in all physical matter. Harris and Crede's book [25] develops this topic at length in several chapters and is considered as a classic on the matter. In the texts on Control and Feedback Systems theory, we find important references about hysteresis. Due to space constraints, we shall only mention two, [26] and [27]. Finally, as our activity is on transducers we shall mention [28], [29] and [30], which deal with materials for sonars.

In electroacoustics we know about performances, which have presumably detected the surround problems mentioned above, and therefore they may even have eliminated them. We can mention the edgless woofer SLE34W and the SLE24W cases manufactured by Fostex. They were units with a double spider. These spiders had quite an important distance between them. In order to avoid the nonlinearities due to the surround that we are dealing with in this article, Fostex removed the suspension, leaving then a minimum distance between the cone and the frame.

2. SOME NONLINEAR RESPONSES

In dynamics, we are going to encounter with what we could call classic nonlinearities, or in other words, nonlinear spectral shapes, or spectral signatures, that are referred to in literature and are well known by many development engineers. We could establish two major groups of nonlinear signatures. The first spectral signature is the best known one. This signature refers to the forced response of systems that show up in all possible distortion forms [12], [13], including the presence of submultiples of the excitation frequencies, or in other words, subharmonics. The other group of nonlinear characteristics is the group that includes all those signatures that come up and appear when trying to know the natural frequencies of the element or system that is being tested. In other words, they are the free response signatures. As these last responses are the ones we are interested in, we shall only work in this specific field.

First, we need to remember the response of linear systems. In order not to be too extensive, we shall simply have at hand all the signatures of a reference element's response, as is the case of a tunefork. Remember that a tunefork will always respond with the same natural frequency and with the same spectral shape whatever the excitation's force amplitude is, and regardless of the type of excitation we use (step, positive, negative, impulse, etc.). Linear systems respond with the same period, no matter what the excitation's amplitude is or what the initial conditions are. Linear systems are isochronous.

It is not so with nonlinear systems, their response is very dependent of the amplitude and of the initial conditions. The theoretical bibliography on the free response of these systems is abundant, we shall only quote a few essential texts: [32], [33], [34] and [35]. From a more experimental point of view, there are other texts where the reader can find experimental works of which we shall only mention [36], [37], [38] and [39]. The books [40] and [41] have an even more experimental charge than the previous texts do.

Reference [42], does not deal specifically with the free response of mechanic systems, but it is nonetheless quoted because it deals with the bilinear support of rotors on the bearings. By presenting this article we pretend to illustrate the complexity of a rotor motion due to the rotor's suspension. We esteem convenient to let the reader see this topic outside the field of loudspeakers. In a very succinct way we mention some articles written by experts who have published researches in the field of nonlinear dynamic from the same point of view we want to give in this paper. Such authors are Rice, Ibrahim, Nayfeh and Balachandran [43], [44], [45], [46] and [47]. An exhaustive list would be too long and rather useless.

3. METHODOLOGY FOR TESTING SURROUNDS AND DIAPHRAGMS

As we are interested in knowing what happens in short strokes and in the main resonance region, we should look for excitation and measuring devices that satisfy our needs. From the measuring point of view, we need a transducer that has very high resolution and does not add any weight to the structure, or very little weight. Later on we shall see we suggest the use of an Eddy Currents Transducer.

From the excitation point of view, it has two aspects, the first is that excitation should not add weight to the testing structure. Initially, we used as an excitation signal the acoustic pressure that came from the impulses generated by a woofer that was placed on the axis of the system to be measured and was placed close to it. Even though the results of this procedure are good, they are not as good as those obtained with the magnetic reluctance driver. Therefore acoustic pressure was withdrawn as the method to excite, and instead we used the reluctance transducer. The second aspect to take into account is that we need to be meticulous with the kind of excitation signal we are going to use, because if we do not choose the appropriate one, we could partially, or completely, loose the spectral signature we are looking for.

As we wish to know the response of a nonlinear system, the use of any kind of excitation signal was not appropriate, nor were appropriate some of the usual signal processing procedures. For example, as we are interested in the "character" or "signature" of the surround, we should not make any spectral, nor any other kind of averages. If it responds in an irregular (nonlinear) way, we should not average these irregularities.

We wish to know the signature of the surround and its associated mass in the region of its eigenvalues. This implies that we want the responses for a sequence of deterministic inputs. As it is possible that there are, and in fact there are, variations in the responses for a sequence of identical force excitations to the sample, we need to know each and every response individually. This is why we work with unique spectrums. In the analyzer we use, which is a 2035 of B&K, each single spectrum is called a Fourier Spectrum.

Usually nonlinear systems have dependencies of the initial conditions. If we use a sinusoidal sweep, we would have the drawback that each state is a function of the previous one. Each time we change the sweep's speed and the input amplitude, we change the initial conditions of the resonance pass by. In other words, for a specific sweep speed, the surround samples analyzed would have a signature, and for a different sweep speed it would, generally, have a different signature. By changing the speed the response changes.

Moreover, each point of the trace of the frequency response is a function of the one that was previously obtained, and this generates an infinite of initial conditions, of impossible interpretation. Despite the many advantages of sinusoidal sweep, especially the low crest factor and the high energy contents, is not advisable for this kind of measurements. The use of random noise is obviously ruled out since this kind of signal usually has a higher level of uncertainty than the one we are looking for here.

Through the average we often reduce or improve the uncertainty, but as we already explained, average is proscribed in this kind of experiences. Consequently, the advisable thing to do is to use an impulse signal and to work at several excitation levels. Therefore, we have worked with discrete pulses that generate flat input spectrums in the frequency range we are interested in. The signal was sent to a reluctance transducer that attracted the diaphragm and the response was measured with an Eddy Current Transducer.

In the analyzer a transient window of the time signal was used to improve the signal to noise ratio. We did so because, at very low signal's levels, noise is significant, and, as the reader knows, impulse signals are one of the worst from this point of view.

Figure 3 is a lay out of the measuring system used in the laboratory for this purpose. The measuring transducer was an Eddy Current displacement transducer manufactured by Bently Nevada, model 3300 XL with an 8 mm head diameter. This transducer is linear in a total stroke of 2 mm. Its sensibility is of 7,87 mV per micrometer. We have placed as a target a steel disc of 25 mm of diameter and 1 mm thick. The disc was properly glued to the diaphragm in a strongly tight way and any local motion was prevented. This target was big enough for the width of the beam the transducer radiates.

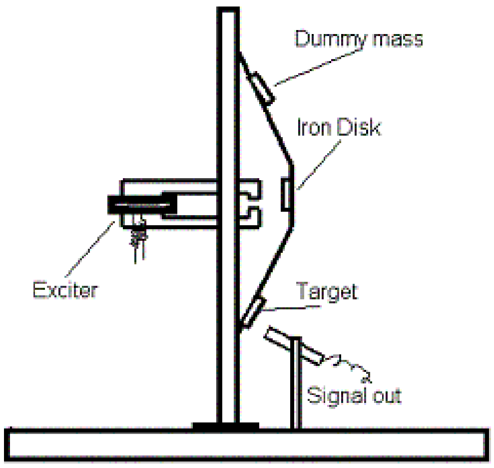

Figure 3: Test bench to evaluate the Suspension-Diaphragm performances. The lay out shows the exciting device, the receiver and the sample being tested

Due to the target's weight, and in order to balance the device, we glued in each diaphragm a symmetric disc that guaranteed the required torque balance within some reasonable margins. Figure 4 shows the details of the target discs' positions. The mass overweight, due to these discs in the diaphragms, has a very low influence in the results. Since motion in the periphery is less regular than that in the axis, we chose for all the tests this area in the diaphragm to take the measurements. Some of these tests' results will demonstrate why.

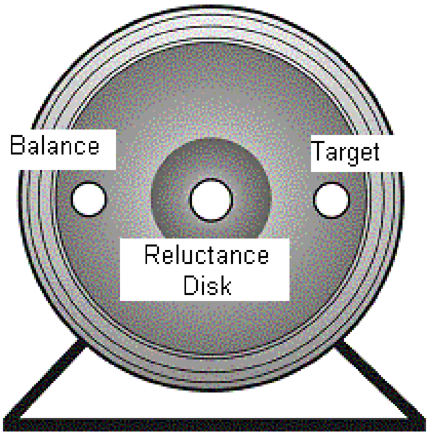

Figure 4: Detail of the target and balance disk, for the diaphragm response measurements by an Eddy current transducer. Notice disks are glued onto the diaphragm and close to the suspension. The high permeability disk for excitation, glued on dust cap, is also depicted

The response-measuring sensor is placed in such a way so that its face is parallel to the target's face. The reader should bear in mind that the diaphragm's axis and the sensor's axis end up forming a determined angle. The values we are going to give are not calibrated in amplitude, but they are proportional between themselves, and the spectral shapes are also correct. The measuring system with this kind of transducer turned out to be of high definition, covering all the strokes the test required. It was also very easy to assemble, to operate, and to adjust.

Let us remember that we chose a reluctance transducer as an exciter, which gave the best input signal repeatability behavior, and had the capability of modifying the force's amplitude when applied to small force ranges. A reluctance transducer is made up of a ferro-magnetic core, which has a coil wrapped around it. The core is open in its outer part (opposite side of the coil) leaving an air gap of 3 mm. This air gap faces the high permeability disc that is firmly glued to the dust cup. The distance between the reluctance transducer and the high permeability disc is chosen according to the force needed and to the maximum stroke that is going to be tested. The high permeability disc attracted by the reluctance transducer is of the same type as the disc we used for the measuring target, that is a 25mm diameter and a 1mm thick.

Because the high-permeability disc is placed in the center of the dust cap, the applied forces will become axial forces. The suspensions under test were glued to the speaker's frame and were firmly fixed by three peripheral points to the test bench, ensuring that the specimen was at rest. Some measurements were taken that ensured that the reluctance sensor's magnetic field did not affect the measuring sensor for all the different circumstances of the tests, which leaves us with no doubts regarding its reliability. Finally, let us remember that the displacement measurement is not an absolute measurement but a relative one measuring the distance between the sensor's head and the target.

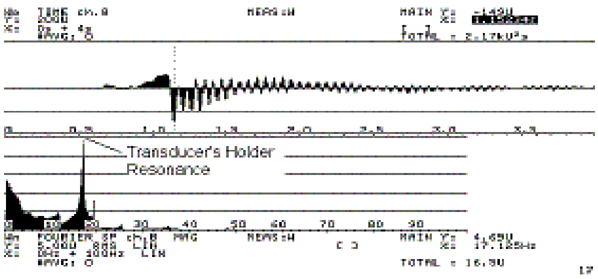

Figure 5: Time signal and Fourier Spectrum of the Transducer's Arm - Support Response to a mechanical pulse. The natural frequency of this transducer and arm at 17.125 Hz is clearly depicted

Generally, the laboratory's background vibration excites the sensor's arm and the sensor. These background motions constitute our measurement floor. To know the natural frequency of the sensor mounted on its arm, we have excited the sensor manually giving motion in its maximum elasticity direction. When these tests were carried out, we placed a heavy target at rest in front of the sensor's head, which we can consider infinite. The time signal and its corresponding Fourier Transform are shown in Figure 5, where we see the main natural frequency of the fixture of our measurement transducer.

In fact, those 17,125 Hz peaks obtained when we analyzed the signals should not be taken in consideration, since they are due to the sensor's base motion. Bear in mind that there can be fluctuations of this frequency when we change the sample because the supporting arm was adapted for each specific sample.

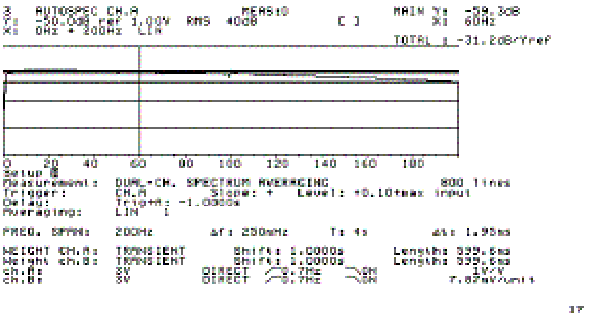

Figure 6: The upper figure depicts the Autospectrum of the electric pulse applied to the power amplifier. The lower figure is one of the most commonly used measurement set-ups for measurements taken up to 200 Hz. For measurements up to 100 Hz, the set up is similar, except for the shift and the length of the transient time window, which is the appropriate

In the upper part of Figure 6 we can observe the Fourier's Transform of the electric pulse that was sent to the power amplifier. Its spectral flatness is sufficient for the needed bandwidth. In the lower part of this figure we can see the measurement set up of those measurements that have a bandwidth of 200 Hz. We placed a Transient Time Window to cut the signal up and only transform that part in which the response is present. It has already been said that the impulsive signal has the drawback that it captures noise before signal is acquired and after it has extinguished, when the signal to noise ratio is low. The rest of implementations are common to any electroacoustics laboratory. All the tests were carried out in a temperature range between 18º and 21ºC.

4. MEASUREMENTS ON SURROUND - DIAPHRAGM SETS

4.1. Sample One: Ten Inches Loudspeaker; Rubber Half Roll Extra Wide Surround

Figures 7, 8 and 9 depict the measurements taken to a rubber half roll sample, its grove had a diameter of 30 mm, that belonged to a 10 inches unit, for three excitations with three different levels. These levels were: very small, small and medium. The figures are explicit showing the time signal and its corresponding Fourier Transforms. When excitation level was very weak, the surround-diaphragm system responded at it's natural frequency of 56.12 Hz. When the stimulus level was increased, then the frequency of the spectral peak fell down to 51.5 Hz. Finally, for medium excitation level the sample gave a value of 54.25 Hz.

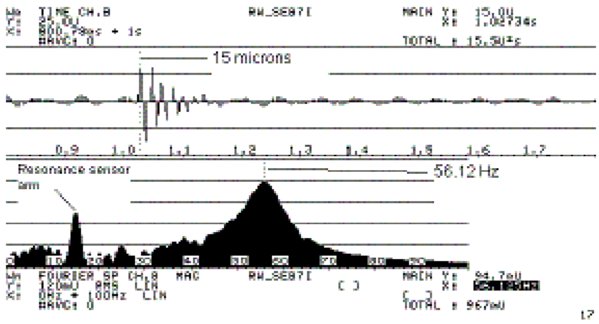

Figure 7: Sample 1: Ten Inches Speaker; Half Roll, Extra Wide (30mm diameter), Rubber Surround. Time signal and Fourier spectrum of the response for a very small force excitation. Maximum time response gives 15 microns

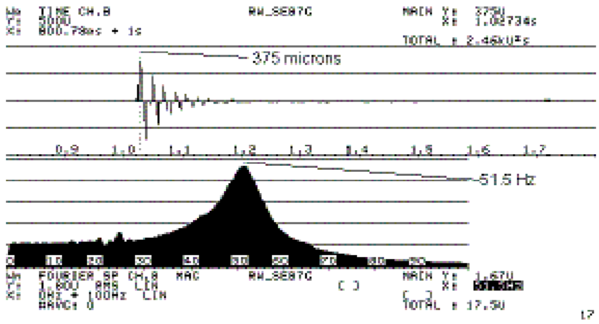

Figure 8: Sample 1: Ten Inches Speaker; Half Roll, Extra Wide (30mm diameter), Rubber Surround. Time signal and Fourier spectrum of the response, for a small – medium excitation level (first positive peak at 375 microns). Notice the shift of the eigenfrequency respect to Figure 7

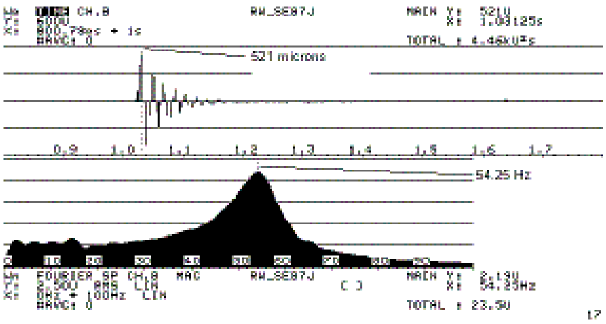

Figure 9: Sample 1: Ten Inches Speaker; Half Roll, Extra Wide (30mm diameter), Rubber Surround. Time signal and Fourier spectrum of the response for a medium excitation (first positive peak at 521 microns). Notice the shift of the eigenfrequency respect to Figures 7 and 8

J. Scott and colleagues [13] found these kinds of results in a three-dimensional graphic when they measured the unit's impedance as a function of the stroke. In this research they tested the whole unit, and so it is not clear what contribution the spider and the surround have on the global behavior. On the other hand, as we shall see, this nonlinear behavior consisting on the variation of the main natural frequency is not unique; there are other nonlinear signatures in suspensions. Summarizing, this sample exhibits a nonlinear signature of the type we could call "main eigenvalue changes obtained from one test to another", or simply, "eigenvalue drift".

4.2. Sample Two: Twelve Inches Loudspeaker; Cloth Three Rolls (Accordion Type) Surround

Bright, [16] is one of the authors who mentioned the effects of rocking in suspended electroacoustic elements, and he does the mechanic approximation the subject requires. The author also mentions the split peak effect we can find in these kinds of structures. The following results correspond to a surround of 3 rolls of plasticized cloth belonging to a speaker of twelve inches of diameter.

In these kinds of surrounds we find two very important characteristics. On one hand we find more linear signatures than with other surrounds, and on the other, the rigid body rotation modes are clearly visible. These rotational modes are oscillations in respect to any of the normal axis, around the axis of symmetry of the suspended diaphragm.

The compromise situation with which design engineers encounter is evident. On one hand, for a surround such as the tested one, if designed and manufactured correctly, generally we can find a rather linear element, but on many occasions it is more likely than others to cause rigid-body diaphragm rotations. Figure 10 shows these secondary peaks, which correspond to a medium level excitation (maximum peak level of the response was 450 microns).

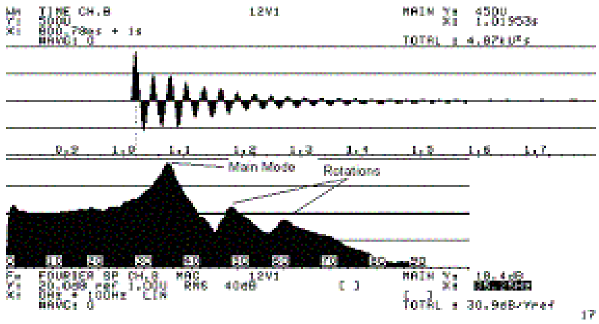

Figure 10: Sample 2:Twelve Inches Speaker; Cloth Three Rolls (Accordion Type) Surround. Time signal and Fourier spectrum of the response. Test at medium level excitation (first positive peak at 450 microns). Notice the secondary peaks the spectrum depicts

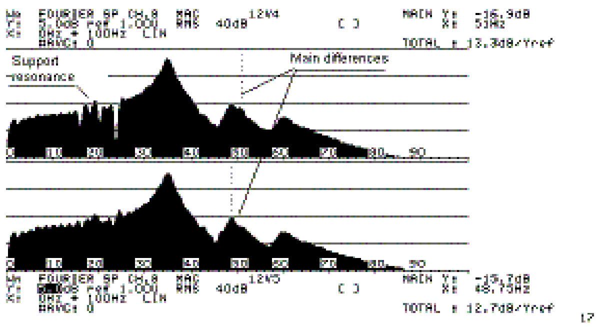

Figure 11: Sample 2: Twelve Inches Speaker; Cloth Three Rolls (Accordion Type) Surround. Fourier spectra of two responses. Test of repeatability for very small stroke. Upper and lower graphics have two different responses for the same input signal. The differences in the spectrum shape are in the secondary peak where the cursor is

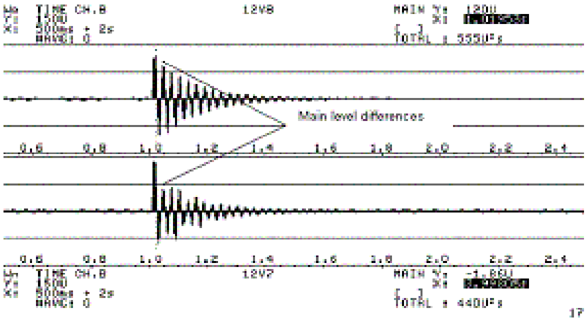

Figure 12: Sample 2:Twelve Inches Speaker; Cloth Three Rolls (Accordion Type) Surround. Time responses of the sample. The tests are low level type (max. peak is 120 microns). Test of response sensitivity to off center forces. The upper curve is the result of an applied force 6 mm at the right of the sample axis. The lower curve is the result of an applied force 6 mm at the left of the sample axis

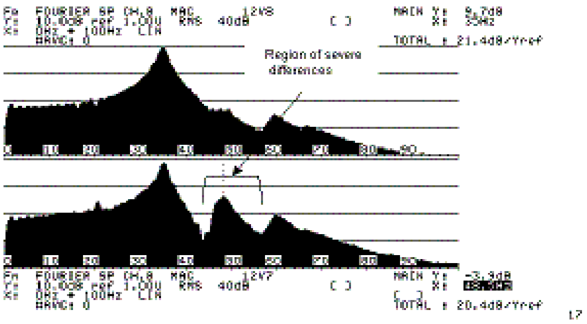

Figure 13: Sample 2: Twelve Inches Speaker; Cloth Three Rolls (Accordion Type) Surround. The graphs are Fourier Transforms of the time signals from Figure 12. Test is of low level type (max. peak is 120 microns). Upper graph depicts the spectrum response for a force applied 6mm to the right away from the axis of symmetry. Lower graph is the spectrum response for a force applied 6mm to the left from the axis of symmetry

When we excite at a very low level we observe that from the measurements repeatability point of view, the system behaves quite linearly. For this sample the only region that is somewhat sensitive to give variations is the one that corresponds with the secondary peaks or the rigid-body rotations. Figure 11 depicts two different spectrum responses for two identical excitations.

The main drawback of this sample of suspension is that probably it does not have a constant stiffness around the full contour. This particular surround sample has a weak point, which is that if we drew the rotary stiffness' trace along the circumference we would find no uniformity. Instead of the straight line it should be we find a function. The sensitivity to the unbalanced force application can be seen in Figures 12 and 13. In these figures it can be observed the responses that the surround-diaphragm gives when the applied force is 6 mm. to the right from the symmetrical axis. It also shows the same process but applying the force 6 mm. to the left of the axis of symmetry.

Time signals present evident differences and its Fourier Transforms are self-explanatory by indicating the surround-diaphragm set's sensitivity to small geometrical variations of the applied force. The spectral differences that Figure 13 shows are outstanding in the spectral area to the upper-right of the natural frequency peak. Observe the marked area in the Spectrum.

This type of surround is more sensitive than others to this irregularity, which consists in the rigidity not being constant through the whole periphery. Notice that the mass unbalance due to the lead wires will tend to cause this kind of motions. Besides these drawbacks, the readers should remember that we might also fall into other mistakes. Mistakes such as using the loudspeaker's frames that are not stiff enough, nor have sufficient mass (infinite frames), or that they present a lack of flatness, or that the face of the box where it is fixed is not flat.

4.3. Sample Three: Ten Inches Loudspeaker; Rubber Half Roll Surround

The sample, the creep of which is shown in Figure 2 with the name of "Rubber", is the one that was tested with the same methodology and the results of which are displayed in the following figures. This sample belongs to a set of 10 inches, and the surround is of half roll with an 18mm diameter.

In Figure 14 we can see two time responses of this set for 99.5 microns and 270 microns responses of maximum peak. The Fourier transforms of these signals are those depicted in Figure 15, where we can observe the natural frequency of the measured sample and the value of which is between 22.5 Hz and 22.7 Hz for these two levels.

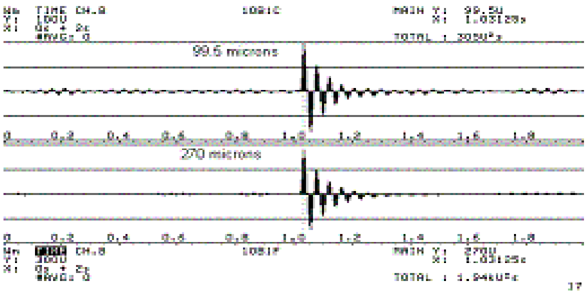

Figure 14: Sample 3: Ten Inches Speaker; Rubber Half Roll Surround. Time Domain responses. The test purpose is to compare the sample for inputs of different levels. The tested amplitudes are: very low level input force (max. peak is 99.5 microns) and low level (maximum peak is 270 microns)

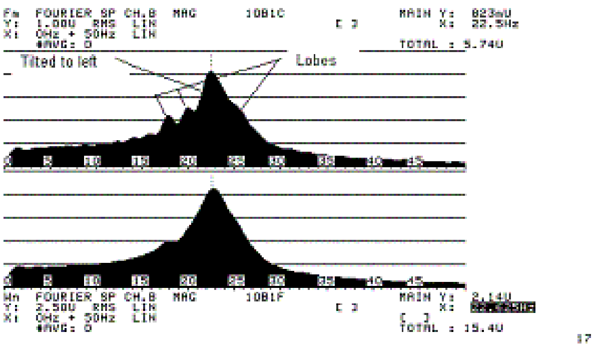

Figure 15: Sample 3: Ten Inches Speaker; Rubber Half Roll Surround. Responses in the frequency domain. Purpose of the test is to compare the responses for different input levels. Tests of very low level type (max. peak is 99.5 microns), and low level (max. Peak is 270 microns). Notice how the spectrum of the smallest response has a "soft type" nonlinear shape

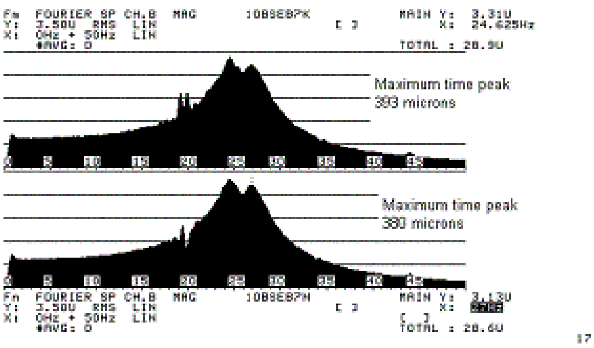

Figure 16: Sample 3: Ten Inches Speaker; Rubber Half Roll Surround. Frequency domain responses for two excitations of very similar input levels. Compare the results with those depicted in Figure 15. Notice the double peaks, or split peaks, and the differences between the two spectra

Notwithstanding, what is most remarkable is that the lower figure looks almost linear in shape whilst the upper figure looks nonlinear despite it being at a lower level than the aforementioned. In the upper figure we see the classical "soft" non linear system, with a smooth slope on the right side of the peak and a strong abrupt slope on the left side. The peak is tilted to the left and has lobes. When referring to the side lower peaks next to the maximum, it seems at first, that we should have some reserves when interpreting them, since they are close to the resonance area of the measuring sensor's fixture and arm. Nevertheless, the tests were repeated in many occasions and the sensor's arm was shortened. The tests were carried out carefully, and the peaks found belong to the specimen tested.

Figure16 depicts the spectral results testing the same sample at higher excitation forces a few days later. Maximum peak of the time response for the upper graphic is 393 microns and for the lower is 380 microns. Observe that the spectral peaks are chopped in both graphs. Because of this slicing on the peaks, there is a high uncertainty of the natural frequency, ranging from 24.62 Hz to 27 Hz (see cursors in the graphs). Notice the high spectral shift of the main natural frequency's position between experiments shown in Figure 15 and Figure 16.

We subjected this sample to a complementary test, trying to test the suspension and come closer to the appearance of the finished product. To do so what we did was to add its coil and leave the spider out. These tests were carried out following exactly the same procedures we have been following through this paper. Naturally, the excitation continues being outside the loudspeaker through the reluctance transducer. The voice coil only acts as an inertial load. The results can be seen in Figure 17.

Although it is represented with a logarithm trace in the vertical axis, its responses are in similar ranges to those in Figure 15 and therefore we could compare them. Observe how we have different responses between them again, and that they are also different in shape to those obtained when the coil was not glued to the diaphragm. Repeatability is very poor for this sample if we take into account that there is a spectral shift of the main natural frequency due to the presence of the coil, which increases the moving part's weight. Observe that in the upper part of figure 17 not even the cursor that corresponds to resonance is in the highest part of the figure. The system responds with a very broad bandwidth.

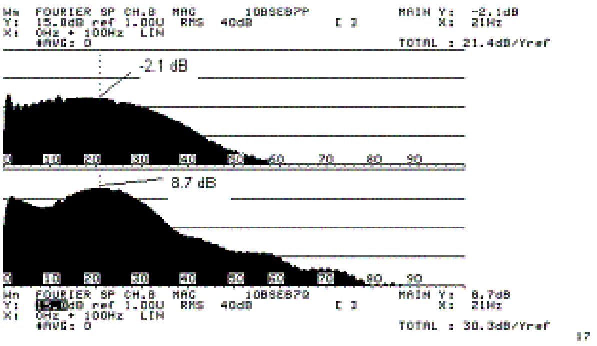

Figure 17: Sample 3: Ten Inches Speaker; Rubber Half Roll Surround. Frequency domain responses of the set with the voice coil glued to the cone. Comparison of the responses for two inputs at different levels

The appearance of the graphs in Figure 17 look more like the forced response of the sample than like the free response we should obtain testing a linear device. But this sample not only shows low repeatability and difficulties to exhibit the main natural frequency, it also offers other characteristics or signatures just as important, such as the splitting or slicing of the main spectral peak. Because the spectral band of maximum response of Figure 17 overlaps with the measuring transducer arm resonance we were careful performing these measurements.

For this sample, everything happens as though the bandwidth of the main peak fluctuated in an important way. Notice that this response is quite far away from the spectral shapes we can find in bibliography. This seems to be logical because the analytical works or the mathematical models referred to damping are much more complex than those on stiffness. Generally, damping is less known both in theory and in practice.

Summing up, this sample presents a very uncommon signature perhaps related with the large creep depicted in Figure 2. We could call this trait "bandwidth variation for repeated inputs". This sample has this trait but also all the other signatures we have observed (shift in natural frequency from test to test, low results repeatability and spectral peak splitting). This sample has both a low main natural frequency and very nonlinear characteristics in the resonance region of its response. It seems as though the viscose contribution has quite a bit of influence or weight in this nonlinear behavior.

4.4. Sample Four: Twelve Inches Loudspeaker; Foam Half Roll Surround

In order to get a better knowledge of the surround behavior for those made of polyurethane we have tested a surround of half roll with a 22 mm diameter belonging to a twelve inches loudspeaker. The tests supply the following results:

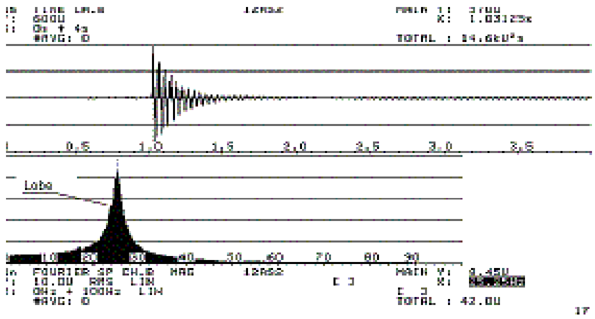

Figure 18: Sample 4: Twelve Inches Speaker; Foam, Half Roll Surround. Time signal and Fourier spectrum of the response for a medium level excitation (first positive time peak at 570 microns). Spectrum peak shows a small lobe on the peak left side

First, it was subjected to a medium level input force giving a response with a maximum positive peak of 570 microns and the spectral shape shows a small lateral left lobe just as seen in Figure 18. Figure 19 has an analogous response level to Figure 18, with a time signal response with a maximum peak of 518 microns and a very symmetrical spectral shape.

So that, comparing both figures for medium level, we see that the sample shows moderate repeatability and moderate nonlinearity. When amplitude decreases, we can observe that our spectral peak is starting to take the appearance of a slightly nonlinear system. Figure 20 shows the response for a maximum positive time peak of 162 microns.

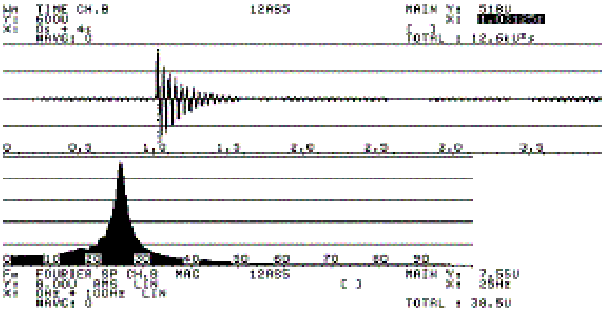

Figure 19: Sample 4: Twelve Inches Speaker; Foam Half Roll Surround. Time signal and Fourier spectrum of the response for a medium level excitation (first positive time peak at 518 microns). Spectrum contour is, almost, perfectly symmetric and smooth in the whole figure

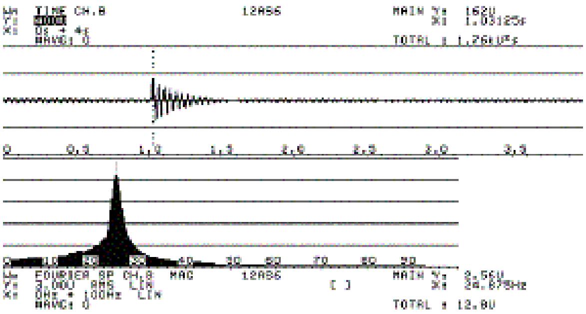

Figure 20: Sample 4: Twelve Inches Speaker; Foam Half Roll Surround. Time signal and Fourier Spectrum for small level of excitation, (first positive time peak at 162 microns). Spectrum peak is becoming asymmetric as input level decreases

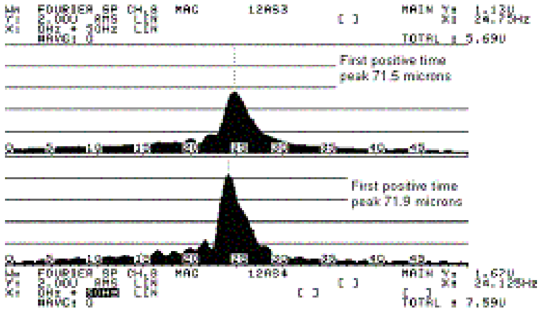

Figure 21: Sample 4: Twelve Inches Speaker; Foam Half Roll Surround. Fourier spectra of the responses for two discrete excitations of very small level. First positive time peak for the upper graph it is 71.5 microns; for the lower graph it is 71.9 microns. Notice the very low repeatability and nonlinear appearance

Still, when we work with very weak levels the nonlinear appearance shapes are much more important. In Figure 21 we can see two spectral shapes corresponding to two discrete responses of which, the maximum positive time peak values are similar.

As the reader may appreciate, the repeatability for very small levels is very low. Besides, the surround and its associated mass exhibit a very clear nonlinearity of the soft type for low test forces.

4.5. Sample Five. Ten Inches Loudspeaker; Cloth Three Rolls (Accordion Type) Surround

A cloth surround of three rolls was also tested in the laboratory. The sample belongs to a 10 inches diameter unit. This sample has given a very linear behavior under the force and temperature conditions in which it was tested. At a medium level of excitation and for a response of which the maximum positive value was of 526 microns, we obtained a very symmetrical spectral shape, as seen in Figure 22.

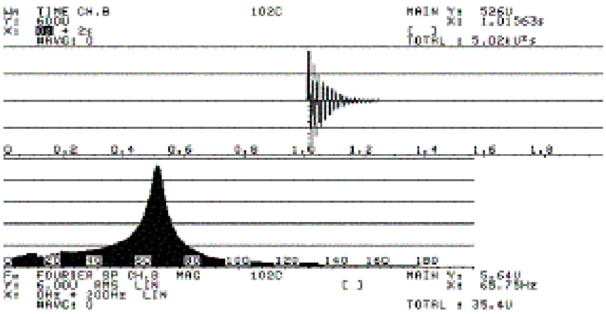

Figure 22: Sample 5: Ten Inches Speaker; Cloth Three Rolls (Accordion Type) Surround.- Test for a discrete medium level excitation. First positive time peak is 526 microns. Notice the remarkable linear appearance

Figure 23 belongs to the spectral responses of two discrete excitations. One of them gave a positive time peak of 108 microns and another of 74.5 microns. Notice the high repeatability of the measurements and the linear shapes of the spectra. This sample was tested even at lower excitation levels, as low as 28 microns maximum positive peak response. The spectral shape continued to be symmetrical and with no noises, nor lobes etc. This result is not shown in order not to lengthen the paper.

Sample five was found to be very linear.

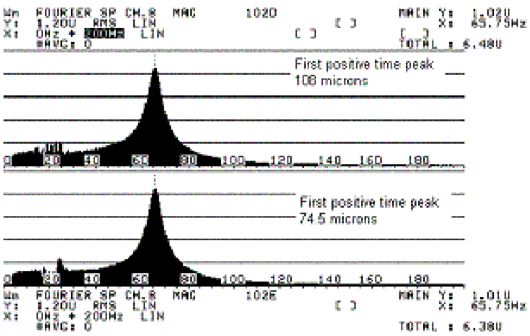

Figure 23: Sample 5: Ten Inches Speaker; Cloth Three Rolls (Accordion Type) Surround. Responses for two discrete excitations. The level of excitations was very small. First positive time peak for upper graph is 108 microns, and for the lower graph it is 74.5 microns. Notice the high repeatability and linear appearance

4.6. Sample Six. Fifteen Inches Loudspeaker; Vulcanized Cloth Asymmetric Surround

We have carried out tests over a larger (diameterwise) unit's surround. For this purpose we used a 15 inches unit. The surround is of vulcanized cloth, formed by two convex half rolls on the radiation's face and one thin convex half roll on the opposite side. The diameter of the convex half rolls on the radiation side is much larger than the diameter on the opposite side's half roll.

The test carried out for very low levels indicates a response whose maximum is in the negative excursion. The response's minimum peak level is of -37 microns, observe the Figure 24. Contrary to other suspensions, the maximum amplitude of this surround is not achieved when it receives the force impulse, but instead when it is free of external forces. The reader can see that, in all the time responses, maximum displacement takes place when the diaphragm is in the backward stroke.

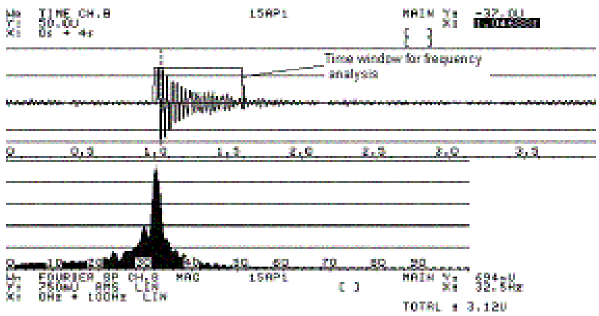

Figure 24: Sample 6: Fifteen Inches Speaker; Vulcanized Cloth Asymmetric Surround. Test for very small input force. Minimum negative time response is 37 microns. The upper graph shows the time window used for this analysis. In the frequency domain notice the lateral lobe at the peak's left side

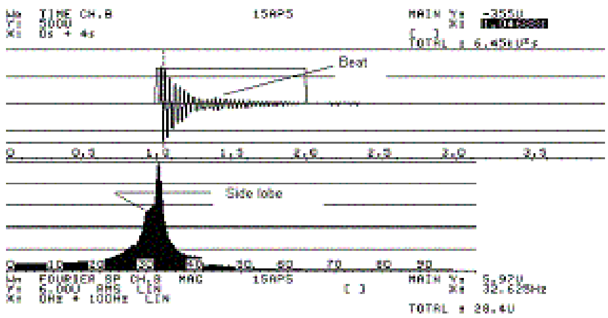

Figure 25: Sample 6: Fifteen Inches Speaker; Vulcanized Cloth Asymmetric Surround. Test for small - medium input force. Minimum negative time response at -355 microns Time window was extended to 1 second. Notice the modulation of the time signal (small beat) and the spectral consequence on the lobe at the left side of the spectrum peak

This surround performs softer when the acting force pushes the diaphragm backward rather than forward. One frequent cause of this behavior is that the system under test exhibits stiffness differences going back and forth.

Devices performing in such a way are, generally, bilinear [42], [48]. Bilinearity in the origin of coordinates, consists of two different slopes for the F(x) curve for both the positive and the negative sense of the diaphragm's displacement. This bilinearity in the origin of coordinates is not shown frequently in stress versus strain experimental graphs, because of the difficulty of measuring very small forces. The time response signature of this surround, with the maximum peak being negative, is probably due to its geometrical asymmetry.

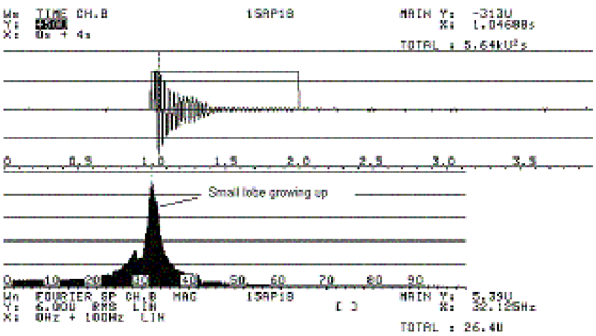

Figure 26: Sample 6: Fifteen Inches Speaker; Vulcanized Cloth Asymmetric Surround. Test for medium level input force. Results after 40 hours of keeping the sample at 8º C. Notice the left lateral spectrum peak now is more independent, and the growing lobe at the right side of the main peak

From the Fourier Transform point of view, we see the tendency to show a small lateral peak which is of smaller frequency than the main one. In Figure 25 we see the same test but applying a higher force giving a minimum time response of -355 microns. In this figure the tendency to find a lobe that will become a lower lateral band is more evident, and this band is the cause of a beat in the time response.

To observe the influence of temperature on the behavior of our sample, we exposed it for 40 hours at a temperature of 8º C, after which was subjected to test. The result is shown in Figure 26, where we can find important spectral variations compared to Figure 25. The most important are two. First, the more distinct separation of the lower lateral peak with a sudden vertical cut on the main peak's left side, and second, an upper lateral lobe gradually starts to appear. We started warming up the sample very slowly to 18º C. This warming up process lasted for seven hours. The results are seen in Figure 27.

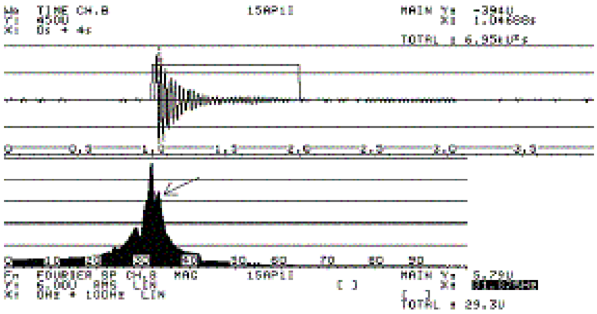

Figure 27: Sample 6: Fifteen Inches Speaker; Vulcanized Cloth Asymmetric Surround.-Test at 18º C, during the suspension's temperature recovery. Observe the evolution of the right side of the spectrum peak

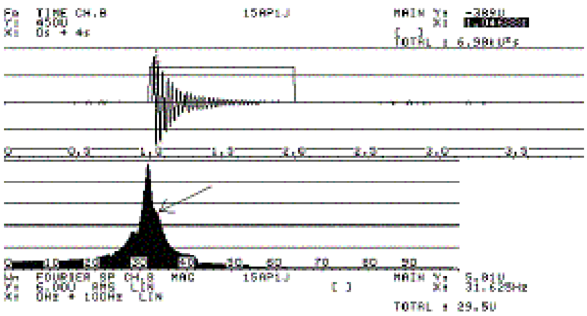

Figure 28: Sample 6: Fifteen Inches Speaker; Vulcanized Cloth Asymmetric Surround.- Test at 19º C,

during the suspension's thermal recovery. Notice the evolution of the right side of the spectrum peak

We continued the warming up process and an hour later the temperature of the sample was of 19º and it then responded to the stimulus as Figure 28 depicts. After observing this figure the reader can see how the diaphragm tends to search for a restitution of the response given in Figure 25. The whole warming up process is not detailed so as not to lengthen this paper too much and we leave it for a later date. The sensitivity to temperature variations and to the "thermal history" of the sample is more imputable to the viscose part than to the compliance part in this sample.

The bilinear stiffness in the origin of coordinates, if confirmed, must be, obviously, of a compliance nature. In this sample both the compliance and the viscose actions are seen. The latter was shown by the temperature change of the sample. The combination of two nonlinearities over the same sample generates a much greater complexity in the interpretation of the results. This is why the restitution of the initial spectral shape (before cooling it) is not easily achieved for this sample.

There are several complementary tests pending. Among them we can highlight: time domain analysis, forced response, and response measurement after fatigue.

5. CONCLUSIONS

The behavior of the transducer in the spectral main natural frequency region is important for the loudspeaker. It is very advisable to design and manufacture transducers as free of nonlinearities at the main natural frequency as possible. The suspension nonlinearities contribute to the final quality of the sound reproduction.

Suspension in a loudspeaker is complex as it consists of the interaction between stiffness and damping in the same physical element, being possible that one of them or both generate nonlinear behaviors. We have offered an advisable methodology to evaluate in the frequency domain surrounds and their associated diaphragms. We suggest first to examine the discrete component on its own.

From the excitation point of view it was very useful to use as an excitation signal impulses fed through a reluctance transducer. From the response point of view this paper recommends Discrete Fourier Transforms (D.F.T.) of the time response (transient windowed) which should be independent and, above all, not averaged.

As a measurement transducer we suggest the use of an Eddy Current Transducer because it provides high definition, it is easy to set up and it is reliable.

We suggest a spectrum signature analysis of the responses to evaluate the nonlinearity levels of the surrounds. We also propose the use of sequences of identical excitations and sequences of scaled excitations in order to observe the nonlinear signatures, the repeatability etc.

All the samples tested have manifested clear and well-differentiated signs. From the six samples tested only one of them, showed for all the tests and at the laboratory's temperature, a linear behavior.

In general, the response is a function of the temperature and of the "thermal history" of the sample. In order to complete our tests it is necessary to experience at several temperatures. The results we present here, except for one done at 8ºC, have only been carried out in the range of temperatures between 18º and 21º C.

As in practice there are almost always certain nonlinear minimums that we cannot elude, we need to develop transducers with the maximum knowledge on the type of nonlinearity associated to the chosen suspension and we also need to know their consequences in the reproduction of sound. It is up to the development engineer's criteria the correspondence of these behaviors with the subjective evaluation of quality in sound reproduction.

If we have a range of classed responses it will be easier to evaluate the compromise situation that we shall adopt when we are selecting a specific component. It is better to have a semi-objective or semi-subjective method, rather than, no method at all.

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