ON THE USE OF THE FORCE DENSITY VECTOR WITHIN ROOM ACOUSTICS

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ABSTRACT

Simple analytical formulas are still of interest for the prediction of basic sound field properties. The conventional calculation scheme based on the sound intensity (IB model) is here overviewed for the most common applications in room acoustics. To refine the predictions in the 3D case an alternative scheme is developed, which is based on geometrical acoustics and on the physical quantity called force density. The sound energy density evaluated by the new method (called FB model) differs from the previous theory based on the sound intensity. As a consequence, the predicted decay curves of the sound energy are different. The consistency of this new method with the available experimental data is investigated.

INTRODUCTION

The fundamental characteristics of sound field in rooms, like sound level and reverberation time, can be usually described by simple formulas within acceptable precision. A more detailed prediction scheme is possible by means of computer models. The geometrical model of image sources, combined with the concept of sound intensity from a single source assumed to act like a point source, can cover a wide range of room acoustical applications and is employed by practitioners [1][2]. This approach, which can be called "intensity based" (IB), provides simplified solutions for typical rooms where axial, tangential and fully tri-dimensional sound fields can be assumed. In this work some of the more important results from the IB model are recalled or re-expressed in closed form and the attention is then focused on the IB 3D case [3]. The application of the IB scheme, though greatly improving Sabine's formulation [4], is not able to make entirely accurate predictions of the sound field characteristics. To challenge this task a new model is presented and discussed which is based on the force density in the sound field (FB model) and not on sound intensity [5]. The merits of the new model are discussed and specific experimental results are presented.

OVERVIEW OF THE IB MODEL FORMULAS

The IB model can be conveniently applied to the calculation of the sound level and of the reverberation time in rooms. What is to be specified prior to using the algorithm is just the geometry of the enclosure under investigation. In fact, by the knowledge of the room dimensions, also the array of the image sources can be set and the mean free path can be calculated. Basically there are four types of enclosures that can be treated analytically with these tools, and a closed solution can be found for each case. The sound fields covered in what follows can be categorised as:

- 1D: this approximation is typical of wide and low rooms where the multiple reflections between floor and ceiling govern the sound field. The image sources lay on an axis perpendicular to the two surfaces. The measurements are usually interested in the points off this axis or, in other words, parallel to the floor and the ceiling (see Fig. 1);
- 2D: this type of sound field is found in rooms where both floor and ceiling are heavily sound absorbing whereas the lateral walls are not. The sound sources are distributed only on a plane including a cross section of the room with the original source. The measurements are done basically on the same plane but within the borders of the room;
- 2D-L: it is a combination of the first two sound fields and is typically found in long rooms. The image sources are distributed as in 2D but the data are sampled like in a 1D field ;
- 3D: this case covers most of ordinary rooms where dimensions are scaled to small integers. The image sources form a tridimensional array and the measurements are usually made in different directions and heights.

Each of the former types of sound fields feature different distribution of image sources and different mean free path. In the Table 1 the values of these quantities are reported.

Sound field	Mean free path (*l*)	Order of reflection (*n*)	N° of sources at given distance (*N*)	Differential N° of sources (*dN*)
1D	H_{1D}	$n_{1D}(y)=\frac{y}{H_{1D}}$	$N_{1D}(y)=\frac{2y}{H_{1D}}$	$dN_{1D}(y)=\frac{2}{H_{1D}}dy$
2D	$\frac{\pi S_{2D}}{L_{2D}}$	$n_{2D}(r)=\frac{L_{2D}r}{\pi S_{2D}}$	$N_{2D}(r)=\frac{\pi r^2}{S_{2D}}$	$dN_{2D}(r)=\frac{2\pi r}{S_{2D}}dr$
2D-L	$\frac{\pi S_{2D}}{L_{2D}}$	$n_{2D-L}(y)=\frac{L_{2D}y}{\pi S_{2D}}$	$N_{2D-L}(y)=\frac{\pi y^2}{S_{2D}}$	$dN_{2D-L}(y)=\frac{2\pi y}{S_{2D}}dy$
3D	$\frac{4V_{3D}}{S_{3D}}$	$n_{3D}(r)=\frac{rS_{2D}}{4V_{3D}}$	$N_{3D}(r)=\frac{4\pi r^3}{3V_{3D}}$	$dN_{3D}(r)=\frac{4\pi r^2}{V_{3D}}dr$

Table 1: The basic geometrical characteristics of typical sound fields of different dimensions.

The subscript "1D", "2D" or "3D" refer to the respective approximation. The surface S_{2D} is the base surface of the room in the 2D case (and L_{2D} is the perimeter of the same surface) whereas S_{3D} is the entire surface of the 3D case. H_{1D} is the height of the "low" room representative of a 1D case. Before presenting the closed solutions for each case it is worth to remind the starting point of the calculation. It is in fact assumed that each image source at distance *r* from the original one emits spherically with sound intensity:

$$\mathrm{I}(r)=\frac{W}{4\pi r^2}(1-\bar{\alpha})^n \qquad (1)$$

where *W* is the sound power and *n* is the order of the image source. The absorption of the air has been neglected on purpose. By simple elaboration one can write:

$$\mathrm{I}(r)=\frac{W}{4\pi r^2}e^{-\gamma r} \quad \text{where} \quad \gamma=-\frac{\ln(1-\bar{\alpha})}{l};\ (\gamma>0) \qquad (2)$$

and *l* is the mean free path. From these simple elements and from the Tab. 1 all of the cases can be solved in closed form. In what follows we will assume that the sound absorption coefficient is constant on the surfaces. For the case of unevenly distributed sound absorption some specific amendments can be introduced.

<u>The 1D Case</u>

As shown in Fig. 1 the geometry for the 1D case is rather special, since the image sources and the measurement points are off axis. This means that the system is conveniently described by two variables, one called *y* that "counts" the image sources and the other called *R* fixing the distance of the receiver from the origin (location of the original source which is assumed at half height of the room). Then the calculation of the total sound intensity collected at the receiver is the incoherent sum of the contributions of all image sources and can be expressed in integral form as:

$$I_{1D}(R)=\frac{W}{4\pi}\int_{\frac{H_{1D}}{2}}^{\infty}\frac{e^{-\gamma_{1D}y}}{R^2+y^2}\frac{2}{H_{1D}}dy \qquad (3)$$

where the lower limit is set at the boundary of the room. In fact off that point there is the “virtual” space where the image sources are collected. This integral can be solved to obtain the total reverberant intensity at the receiver:

$$I_{1D}(R)=\frac{W}{4\pi H_{1D}R}\{2CosInt(\gamma_{1D}R)Sin(\gamma_{1D}R)+Cos(\gamma_{1D}R)[\pi-2SinInt(\gamma_{1D}R)]-f(H_{1D},\gamma_{1D})\}$$

$$f(H_{1D},\gamma_{1D})\rightarrow 0 \ for\ H_{1D}\rightarrow 0\,. \qquad (4)$$

The solution is easily implemented since the required functions (whose definition is found in [6]) are available in most calculus software. It is to be noted that this is only the so-called “specular” component derived firstly in series expansion by Sacerdote [7]. Unfortunately the “diffused” (or scattered) part requires a much more difficult treatment [8].

The 2D Case

In this case the solution applies for receivers laying approximately on the same plane which includes the original sound source and its replicas. The integral solving the problem is:

$$I_{2D}(r)=\frac{W}{2S_{2D}}\int_{r}^{\infty}\frac{e^{-\gamma_{2D}x}}{x^2}xdx=\frac{W}{2S_{2D}}\int_{\gamma_{2D}r}^{\infty}\frac{e^{-\delta}}{\delta}d\delta \qquad (5)$$

where δ is an integrating variable and the lower limit is the receiver position. Also in this case the solution for the reverberant field can be expressed in closed form by a known function:

$$I_{2D}(r)=\frac{W}{2S_{2D}}ExpInt(\gamma_{2D}r)\,. \qquad (6)$$

The 2D-L Case

As reported above this case has features of both preceding cases. In fact the variables are chosen according to the 1D case whereas the mean free path is typically from the 2D case. In this case the lower limit y_0 can be set to an average of the distances of the sound source from the lateral walls. It reads:

$$I_{2D}(R)=\frac{W}{2S_{2D}}\int_{y_0}^{\infty}\frac{e^{-\gamma_{2D}y}}{R^2+y^2}ydy\,. \qquad (7)$$

The solution has evident similarities with that of the 1D case but now the dependence 1/R has disappeared:

$$I_{2D-L}(R)=\frac{W}{4S_{2D}}\{Sin(\gamma_{2D}R)[\pi-2SinInt(\gamma_{2D}R)]-2Cos(\gamma_{2D}R)CosInt(\gamma_{2D}R)-g(y_o,\gamma_{2D})\}$$

$$g(y_0,\gamma_{2D})\rightarrow 0 \ for\ y_0\rightarrow 0\,. \qquad (8)$$

The 3D Case

We repeat the exercise in the 3D case. After simple calculation the well-known formula by Barron is obtained:

$$I_{3D}(r)=\frac{W}{4\pi}\int_{r}^{\infty}\frac{e^{-\gamma_{3D}x}}{x^2}dN_{3D}(x)=\frac{W}{V_{3D}\gamma_{3D}}\int_{\gamma_{3D}r}^{\infty}e^{-\delta}d\delta=\frac{W}{V_{3D}\gamma_{3D}}e^{-\gamma_{3D}r}\,. \qquad (9)$$

This formula predicts a pure exponential decay of reverberant energy both in time and space. It is to be noted that in the integral the lower limit is set again at the receiver. The limit can be set to a fixed value like the mean free path (and in this case the level does not depend anymore from *r*), or to the extra distance travelled by the first reflection. Another option is *r*=0 and in this last case [4] one obtains a useful modification Sabine’s formula for the constant reverberant energy, reading:

$$I_{3D}(0) = \frac{W}{V_{3D}g_{3D}} \int_0^{\infty} e^{-d} dd = \frac{W}{V_{3D}g_{3D}} = -\frac{4W}{S_{3D}\ln(1-\overline{a})}. \qquad (10)$$

SOME OPEN PROBLEMS

Now the attention can be focused on the last, 3D, case which has vast applications by practitioners. Eq. (9) with addition of the direct sound has proved basic agreement with experiment. This is valid both for the prediction of sound level through the Barron formula and for the reverberation time according to the Eyring formula, which can be easily derived in the time domain with the same equation. Though the agreement with experiment is generally satisfying there are still some points which call for an improvement even in the most simple 3D cases. In particular the prediction of the sound level can be overestimated at farthest locations as represented for instance in Fig. 2 for a typical shoebox-shaped auditorium where the 3D IB model has been applied. In this room the discrepancy is on average around 1dB at rear seats. Moreover by means of IB 3D model one single figure is obtained for the reverberation time in the whole enclosure. This fact stems from the exponential course of the decay of sound energy in the room: the decay constant is considered unique and the same decay represents the whole enclosure. Again this seems not entirely descriptive of the real physical situation, since a weak but notable increase of reverberation time with distance from the sound source can be detected. As an example Fig. 3 shows the dependence of the measured parameter RT30 in the same hall as before. The increase, though small, is quantified by linear regression. This evidence is more pronounced in highly sound absorptive enclosures [5]. From the theoretical point of view this means that the energy decay curve, or, in other words, the integrated impulse response known among practitioners as “Schroeder integral”, is not exactly an exponential. This is valid for the reverberant part of the decay, that is, when taking away the direct sound. These facts can actually be critical in some respects and prompt for a different approach to the problem.

THE FB MODEL

The basic idea underlying the new model is to go back to the starting point of the algorithm and use a different physical quantity, the force density, instead of the sound intensity. Within this model every image source applies a force vector at the receiver. All of the force contributions from the pattern of image sources are thus composed in a global force as done in mechanics. At last it is necessary to compute the work of this global force and the sound energy is obtained. After this quick sketch of the scheme, whose details are expounded in [5], it is necessary to introduce the force density vector. This vector can be defined after the work of Morse and Ingard [9] and with regards to the variable *r*, one gets

$$\mathbf{f} = -\frac{1}{c}\frac{\partial \mathbf{I}}{\partial r}. \qquad (11)$$

By means of Eq. (11) it is possible to calculate the force density vector once the sound intensity vector is known. This definition is completely general and no hypothesis is necessary except that of the linear solution of the wave equation. The computation can thus be accomplished for the 3D case. The other input data are the uniform sound absorption as above and the geometry of image sources described in the last row of Tab. 1 under the “3D” case. After some laborious passages (reported in [5]) the following result is obtained:

$$E_{3D}(r) = \frac{W}{3cV_{3D}}\frac{e^{-g_{3D}r}}{g_{3D}} + \frac{2}{3g_{3D}}\int_{g_{3D}r}^{\infty} ExpInt(x)\,dx. \qquad (12)$$

It is to be noted that here we properly speak of sound energy, since this is the work of a force, whereas in the IB cases the physical dimensions of energy are obtained after dividing by c, the sound velocity.

DISCUSSION

Firstly the first term on the right of Eq.(12) has an exponential course and formally coincides with the IB result, apart a 1/3 factor coming from a space average performed in the FB model (not reported here). But the main interesting fact is the second term involving the integral of the exponential-integral function. This function can still be implemented without excessive troubles in most current software for calculus and has not and exponential course. If one examines Eq. (12) in the time domain this term contributes especially in the former part of the decay of sound energy. This concept is explained in Fig. 4 where the integrated impulse responses for the purely exponential IB model and for the FB model are reported. In the figure the two terms of Eq. (12) are also traced separately. The overall course of the FB model is not exponential and presents a slight concavity in the first part, where the second term is more relevant. This justifies the weak increase of reverberation time with increasing distance from the source. In fact, when moving away from the sound source, curves with decreasing concavity are obtained. This provides the slight increase in the reverberation time. Moreover, compared to the IB model, a different reverberation time is obtained if the same averaged absorption is fixed. This has strong implication in the prediction of reverberation time and a revision of the current formulas can be proposed [5]. Finally also the prediction of the sound level is improved as verified in the Fig. 2 where also the curve obtained form the FB model was plot. The improvement is remarkable especially at rear positions in the hall.

CONCLUSIONS

After an overview of the current resources for easy prediction of the sound decay in enclosures by means of the IB model, some open problems with the 3D case were reported. A novel calculation scheme was developed with the aim on the one side not to require complicated computer design (al least at the level of basic geometry) and, on the other, to improve and possibly resolve the open points recalled for the IB model. The new model, called FB or force based model, rests on the same geometrical assumptions of the IB model. The application to the 3D case has evidenced some good properties:

- a mixed type of decay of sound energy causes a slight concavity in the initial part of the decay curve ;
- compared to the IB model the predictions of sound level are improved;
- for fixed sound absorption values the estimate of reverberation time are lower than those obtained either with the Sabine or Eyring formulae;
- the FB model is able to estimate a slight increase in the reverberation time along with an increased distance from the sound source.

More work is needed to implement the FB model in the other sound fields (namely 1D to 2D-L).

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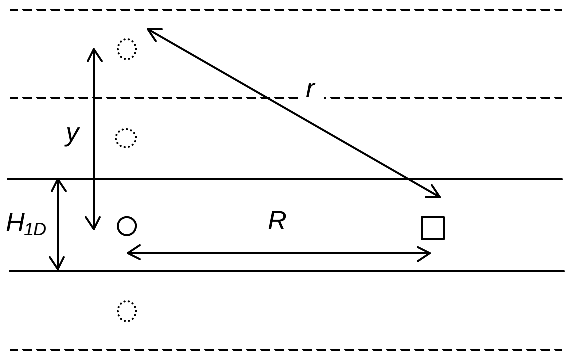

Fig. 1 - Sketch of the geometry and choice of variables in the 1D sound field. The image sources extend over and below towards infinity.

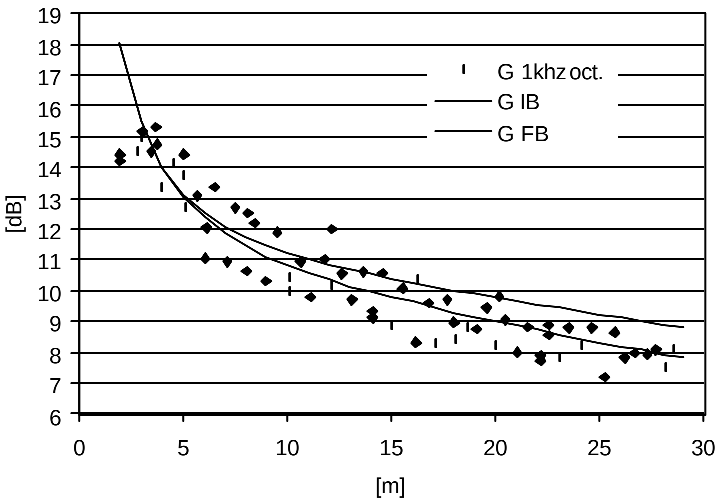

Fig. 2 (after [5]) -Decrease of sound level with increasing distance form the sound source expressed in terms of strength G. Comparison of measured values with IB and FB curves. The hall is shoebox-shaped with 4000m^3 volume.

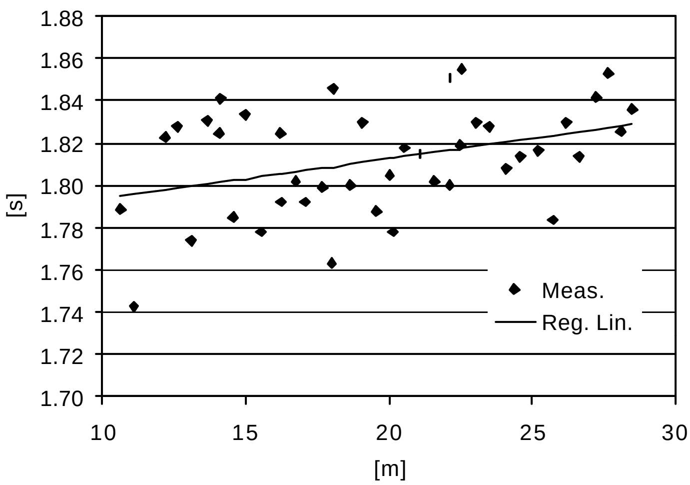

Fig. 3 - The slight increase in RT measured in the same hall as in Fig. 2. The linear regression line of data is also included.

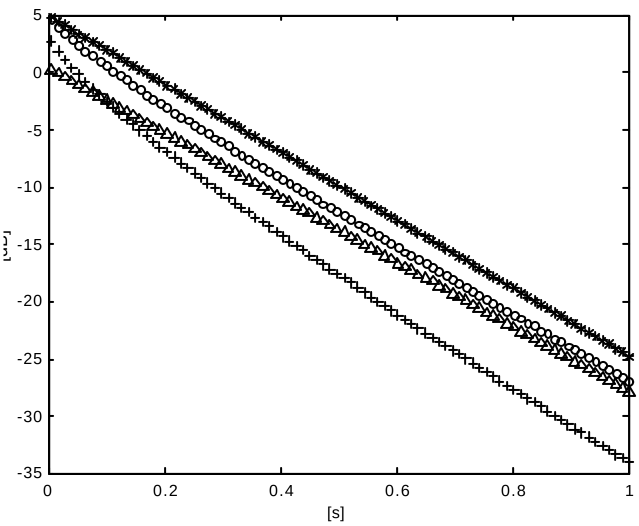

Fig. 4 (after [5]) - Prototype hall having a shoebox shape and overall dimensions of 20x20x50m. Plots of Schroeder for the reverberating parts of sound energy: non exponential part of FB model (+), exponential part of FB model (Δ), global FB model (O) and IB model (*).