Introduzione

Ho deciso di scrivere le seguenti tavole, le quali mi auguro saranno sempre in continua crescita, allo scopo di sostenere i fisici nel loro continuo bisogno di fare conti senza scassarsi le palle, i miei amici e me stesso in primis. Tutto il materiale qui raccolto non é altro che (in minima misura) una rielaborazione di contenuti tratti da Wikipedia (cosí Chiacchera é contento). Solo un ultimo consiglio all'utente:

Usatemi ma non **Ab**usatemi

Pietro Doná

Regole di Integrazione generali e funzioni elementari

Regole per l'integrazione di funzioni semplici

$$\int af(x)\,dx = a\int f(x)\,dx$$
$$\int [f(x)+g(x)]\,dx = \int f(x)\,dx + \int g(x)\,dx$$
$$\int f(x)g(x)\,dx = f(x)\int g(x)\,dx - \int \left(d[f(x)]\int g(x)\,dx\right)dx$$

Regole per l'integrazione di funzioni razionali

$$\int dx = x + C$$
$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C \qquad \text{per } n \neq -1$$
$$\int \frac{1}{x}\,dx = \ln|x| + C$$
$$\int \frac{1}{1+x^2}\,dx = \arctan x + C$$

Regole per l'integrazione di logaritmi

$$\int \ln x\,dx = x\ln x - x + C$$
$$\int \log_b x\,dx = x\log_b x - x\log_b e + C$$

Regole per l'integrazione di funzioni esponenziali

$$\int e^x\,dx = e^x + C$$
$$\int a^x\,dx = \frac{a^x}{\ln a} + C$$

Regole per l'integrazione di funzioni irrazionali

$$\int \frac{1}{\sqrt{1-x^2}}\,dx = \arcsin x + C$$
$$\int \frac{-1}{\sqrt{1-x^2}}\,dx = \arccos x + C$$
$$\int \frac{x}{\sqrt{x^2-1}}\,dx = \operatorname{arcsec} x + C$$
$$\int \frac{1}{\sqrt{1+x^2}}\,dx = arc\sinh x + C$$

$$\int \frac{1}{\sqrt{x^2-1}}\,dx = arc\cosh x + C$$

$$\int \sqrt{a^2-x^2}\,dx = \frac{a^2}{2}\arcsin\frac{x}{a} + \frac{x}{2}\sqrt{a^2-x^2} \ + C$$

Regole per l'integrazione di funzioni Trigonometriche

$$\int \cos x\,dx = \sin x + C$$

$$\int \sin x\,dx = -\cos x + C$$

$$\int \tan x\,dx = -\ln|\cos x| + C$$

$$\int \csc x\,dx = \ln|\csc x - \cot x| + C$$

$$\int \sec x\,dx = \ln|\sec x + \tan x| + C$$

$$\int \cot x\,dx = \ln|\sin x| + C$$

$$\int \sec^2 x\,dx = \tan x + C$$

$$\int \csc^2 x\,dx = -\cot x + C$$

$$\int \sin^2 x\,dx = \frac{1}{2}(x - \sin x\cos x) + C$$

$$\int \cos^2 x\,dx = \frac{1}{2}(x + \sin x\cos x) + C$$

Regole per l'integrazione di funzioni iperboliche

$$\int \sinh x\,dx = \cosh x + C$$

$$\int \cosh x\,dx = \sinh x + C$$

$$\int \tanh x\,dx = \ln(\cosh x) + C$$

$$\int \operatorname{csch} x\,dx = \ln\left|\tanh\frac{x}{2}\right| + C$$

$$\int \operatorname{sech} x\,dx = \arctan(\sinh x) + C$$

$$\int \coth x\,dx = \ln|\sinh x| + C$$

$$\int \operatorname{arccosh} x\,dx = x\operatorname{arccosh} x - \sqrt{x^2-1} + C$$

$$\int \operatorname{arcsinh} x\,dx = x\operatorname{arcsinh} x - \sqrt{x^2+1} + C$$

$$\int \operatorname{arctanh} x\,dx = x\operatorname{arctanh} x + \frac{\log(1-x^2)}{2} + C$$

Integrali definiti

Alcuni Integrali definiti "elementari"

$$\int_0^{+\infty} \sqrt{x}\, e^{-x}\, dx = \frac{1}{2}\sqrt{\pi}$$

$$\int_0^{+\infty} e^{-\frac{x^2}{2}}\, dx = \sqrt{\frac{\pi}{2}} \text{ (integrale di Gauss)}$$

$$\int_{-\infty}^{+\infty} e^{\frac{-u^2}{2}} du = \sqrt{2\pi} \text{ (Integrale di Eulero)}$$

$$\int_0^{+\infty} e^{-x^2}\, dx = \frac{1}{2}\sqrt{\pi}$$

$$\int_0^{+\infty} \frac{x}{e^x - 1}\, dx = \frac{\pi^2}{6}$$

$$\int_0^{+\infty} \frac{x^3}{e^x - 1}\, dx = \frac{\pi^4}{15}$$

$$\int_0^{+\infty} \frac{\sin(x)}{x}\, dx = \frac{\pi}{2}$$

$$\int_0^{+\infty} x^{z-1}\, e^{-x}\, dx = \Gamma(z) \text{ (Γ; denota la funzione Gamma)}$$

$$\int_0^{1} \frac{1}{\sqrt{1-t^3}}\, dt = \frac{1}{3}\beta\left(\frac{1}{3}, \frac{1}{2}\right) \text{ (integrale ellittico) $\beta\,(p,q)$ denota la funzione Beta}$$

$$\int_0^{\frac{\pi}{2}} \ln(\cos(x))\, dx = \int_0^{\frac{\pi}{2}} \ln(\sin(x))\, dx = -\frac{\pi}{2}\ln(2)$$

$$\int_{-\infty}^{+\infty} \frac{\sin\, x}{x}\, dx = \pi$$

$$\int_{-\infty}^{+\infty} \cos(x^2)\, dx = \int_{-\infty}^{+\infty} \sin(x^2)\, dx = \sqrt{\frac{\pi}{2}} \text{ (integrali di Fresnel)}$$

$$\int_0^{\pi} \ln(1 - 2\alpha\cos\, x + \alpha^2)\, dx = 2\pi \ln|\alpha|$$

$$\int_0^{+\infty} x e^{-x^3}\, dx = \frac{1}{3}\Gamma\left(\frac{2}{3}\right)$$

Funzioni speciali da integrali trigonometrici e iperbolici

$$\mathrm{Si}(x) = \int_0^x \frac{\sin t}{t}\, dt$$

$$\mathrm{si}(x) = -\int_x^{\infty} \frac{\sin t}{t}\, dt = \mathrm{Si}(x) - \frac{1}{2}\pi$$

$$\mathrm{Ci}(x) = \gamma + \ln x + \int_0^x \frac{\cos t - 1}{t}\, dt$$

$$\mathrm{Cin}(x) = \int_0^x \frac{1 - \cos t}{t}\, dt$$

$$\mathrm{ci}(x) = -\int_x^{\infty} \frac{\cos t}{t}\, dt$$

$$\mathrm{Shi}(x) = \int_0^x \frac{\sinh t}{t}\, dt = \mathrm{shi}(x)$$

$$\mathrm{Chi}(x) = \gamma + \ln x + \int_0^x \frac{\cosh t - 1}{t}\, dt = \mathrm{chi}(x)$$

Integrali indefiniti di funzioni razionali

$$\int (ax+b)^n dx = \frac{(ax+b)^{n+1}}{a(n+1)} \quad (\text{per } n \neq -1)$$

$$\int \frac{dx}{ax+b} = \frac{1}{a}\ln|ax+b|$$

$$\int x(ax+b)^n dx = \frac{a(n+1)x-b}{a^2(n+1)(n+2)}(ax+b)^{n+1} \quad (\text{per } n \notin \{-1,-2\})$$

$$\int \frac{x\ dx}{ax+b} = \frac{x}{a} - \frac{b}{a^2}\ln|ax+b|$$

$$\int \frac{x\ dx}{(ax+b)^2} = \frac{b}{a^2(ax+b)} + \frac{1}{a^2}\ln|ax+b|$$

$$\int \frac{x\ dx}{(ax+b)^n} = \frac{a(1-n)x-b}{a^2(n-1)(n-2)(ax+b)^{n-1}} \quad (\text{per } n \notin \{-1,-2\})$$

$$\int \frac{x^2\ dx}{ax+b} = \frac{1}{a^3}\left(\frac{(ax+b)^2}{2} - 2b(ax+b) + b^2\ln|ax+b|\right)$$

$$\int \frac{x^2\ dx}{(ax+b)^2} = \frac{1}{a^3}\left(ax+b-2b\ln|ax+b| - \frac{b^2}{ax+b}\right)$$

$$\int \frac{x^2\ dx}{(ax+b)^3} = \frac{1}{a^3}\left(\ln|ax+b| + \frac{2b}{ax+b} - \frac{b^2}{2(ax+b)^2}\right)$$

$$\int \frac{x^2\ dx}{(ax+b)^n} = \frac{1}{a^3}\left(-\frac{1}{(n-3)(ax+b)^{n-3}} + \frac{2b}{(n-2)(a+b)^{n-2}} - \frac{b^2}{(n-1)(ax+b)^{n-1}}\right) \quad (\text{per } n \notin \{1,2,3\})$$

$$\int \frac{dx}{x(ax+b)} = -\frac{1}{b}\ln\left|\frac{ax+b}{x}\right|$$

$$\int \frac{dx}{x^2(ax+b)} = -\frac{1}{bx} + \frac{a}{b^2}\ln\left|\frac{ax+b}{x}\right|$$

$$\int \frac{dx}{x^2(ax+b)^2} = -a\left(\frac{1}{b^2(ax+b)} + \frac{1}{ab^2x} - \frac{2}{b^3}\ln\left|\frac{ax+b}{x}\right|\right)$$

$$\int \frac{dx}{x^2+a^2} = \frac{1}{a}\arctan\frac{x}{a}$$

$$\int \frac{dx}{x^2-a^2} = -\frac{1}{a}\operatorname{artanh}\frac{x}{a} = \frac{1}{2a}\ln\frac{a-x}{a+x} \quad (\text{per } |x|<|a|)$$

$$\int \frac{dx}{x^2-a^2} = -\frac{1}{a}\operatorname{arcoth}\frac{x}{a} = \frac{1}{2a}\ln\frac{x-a}{x+a} \quad (\text{per } |x|>|a|)$$

$$\int \frac{dx}{ax^2+bx+c} = \frac{2}{\sqrt{4ac-b^2}}\arctan\frac{2ax+b}{\sqrt{4ac-b^2}} \quad (\text{per } 4ac-b^2>0)$$

$$\int \frac{dx}{ax^2+bx+c} = -\frac{2}{\sqrt{b^2-4ac}}\operatorname{artanh}\frac{2ax+b}{\sqrt{b^2-4ac}} = \frac{1}{\sqrt{b^2-4ac}}\ln\left|\frac{2ax+b-\sqrt{b^2-4ac}}{2ax+b+\sqrt{b^2-4ac}}\right| \quad (\text{per } 4ac-b^2<0)$$

$$\int \frac{x\ dx}{ax^2+bx+c} = \frac{1}{2a}\ln\left|ax^2+bx+c\right| - \frac{b}{2a}\int\frac{dx}{ax^2+bx+c}$$

$$\int \frac{mx+n}{ax^2+bx+c}dx = \frac{m}{2a}\ln\left|ax^2+bx+c\right| + \frac{2an-bm}{a\sqrt{4ac-b^2}}\arctan\frac{2ax+b}{\sqrt{4ac-b^2}} \quad (\text{per } 4ac-b^2>0)$$

$$\int \frac{mx+n}{ax^2+bx+c}dx = \frac{m}{2a}\ln\left|ax^2+bx+c\right| + \frac{2an-bm}{a\sqrt{b^2-4ac}}\operatorname{artanh}\frac{2ax+b}{\sqrt{b^2-4ac}} \quad (\text{per } 4ac-b^2<0)$$

$$\int \frac{dx}{(ax^2+bx+c)^n} = \frac{2ax+b}{(n-1)(4ac-b^2)(ax^2+bx+c)^{n-1}} + \frac{(2n-3)2a}{(n-1)(4ac-b^2)}\int\frac{dx}{(ax^2+bx+c)^{n-1}}$$

$$\int \frac{x\ dx}{(ax^2+bx+c)^n} = \frac{bx+2c}{(n-1)(4ac-b^2)(ax^2+bx+c)^{n-1}} - \frac{b(2n-3)}{(n-1)(4ac-b^2)}\int\frac{dx}{(ax^2+bx+c)^{n-1}}$$

$$\int \frac{dx}{x(ax^2+bx+c)} = \frac{1}{2c}\ln\left|\frac{x^2}{ax^2+bx+c}\right| - \frac{b}{2c}\int\frac{dx}{ax^2+bx+c}$$

Integrali indefiniti di funzioni irrazionali

$$\int \sqrt{a^2-x^2}\,dx = \frac{1}{2}\left(x\sqrt{a^2-x^2}+a^2\arcsin\frac{x}{a}\right) \quad (|x|\le|a|)$$

$$\int x\sqrt{a^2-x^2}\,dx = -\frac{1}{3}\sqrt{(a^2-x^2)^3} \quad (|x|\le|a|)$$

$$\int \frac{\sqrt{a^2-x^2}\,dx}{x} = \sqrt{a^2-x^2}-a\ln\left|\frac{a+\sqrt{a^2+x^2}}{x}\right| \quad (|x|\le|a|)$$

$$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} \quad (|x|\le|a|)$$

$$\int \frac{x^2\,dx}{\sqrt{a^2-x^2}} = -\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\arcsin\frac{x}{a} \quad (|x|\le|a|)$$

$$\int \sqrt{x^2+a^2}\,dx = \frac{1}{2}\left(x\sqrt{x^2+a^2}+a^2\,\mathrm{arsinh}\frac{x}{a}\right)$$

$$\int x\sqrt{x^2+a^2}\,dx = \frac{1}{3}\sqrt{(x^2+a^2)^3}$$

$$\int \frac{\sqrt{x^2+a^2}\,dx}{x} = \sqrt{x^2+a^2}-a\ln\left|\frac{a+\sqrt{x^2+a^2}}{x}\right|$$

$$\int \frac{dx}{\sqrt{x^2+a^2}} = \mathrm{arsinh}\frac{x}{a} = \ln\left|x+\sqrt{x^2+a^2}\right|$$

$$\int \frac{x\,dx}{\sqrt{x^2+a^2}} = \sqrt{x^2+a^2}$$

$$\int \frac{x^2\,dx}{\sqrt{x^2+a^2}} = \frac{x}{2}\sqrt{x^2+a^2}-\frac{a^2}{2}\,\mathrm{arsinh}\frac{x}{a} = \frac{x}{2}\sqrt{x^2+a^2}-\frac{a^2}{2}\ln\left|x+\sqrt{x^2+a^2}\right|$$

$$\int \frac{dx}{x\sqrt{x^2+a^2}} = -\frac{1}{a}\,\mathrm{arsinh}\frac{a}{x} = -\frac{1}{a}\ln\left|\frac{a+\sqrt{x^2+a^2}}{x}\right|$$

$$\int \sqrt{x^2-a^2}\,dx = \frac{1}{2}\left(x\sqrt{x^2-a^2}\mp a^2\,\mathrm{arcosh}\left|\frac{x}{a}\right|\right) \quad (\text{per } |x|\ge\||a|;\ -\ \text{per } x>0,\ +\ \text{per } x<0)$$

$$\int x\sqrt{x^2-a^2}\,dx = \frac{1}{3}\sqrt{(x^2-a^2)^3} \quad (\text{per } |x|\ge|a|)$$

$$\int \frac{\sqrt{x^2-a^2}\,dx}{x} = \sqrt{x^2-a^2}-a\arccos\frac{a}{x} \quad (\text{per } |x|\ge|a|)$$

$$\int \frac{dx}{\sqrt{x^2-a^2}} = \mathrm{arcosh}\frac{x}{a} = \ln\left(|x|+\sqrt{x^2-a^2}\right) \quad (\text{per } |x|>|a|)$$

$$\int \frac{x\,dx}{\sqrt{x^2-a^2}} = \sqrt{x^2-a^2} \quad (\text{per } |x|>|a|)$$

$$\int \frac{x^2\,dx}{\sqrt{x^2-a^2}} = \frac{x}{2}\sqrt{x^2-a^2}+\frac{a^2}{2}\,\mathrm{arcosh}\left|\frac{x}{a}\right| = \frac{1}{2}\left(x\sqrt{x^2-a^2}+a^2\ln\left(|x|+\sqrt{x^2-a^2}\right)\right) \quad (\text{per } |x|>|a|)$$

$$\int \frac{dx}{\sqrt{ax^2+bx+c}} = \frac{1}{\sqrt{a}}\ln\left|2\sqrt{a(ax^2+bx+c)}+2ax+b\right| \quad (\text{per } a>0)$$

$$\int \frac{dx}{\sqrt{ax^2+bx+c}} = \frac{1}{\sqrt{a}}\,\mathrm{arsinh}\frac{2ax+b}{\sqrt{4ac-b^2}} \quad (\text{per } a>0,\ 4ac-b^2>0)$$

$$\int \frac{dx}{\sqrt{ax^2+bx+c}} = \frac{1}{\sqrt{a}}\ln|2ax+b| \quad (\text{per } a>0,\ 4ac-b^2=0)$$

$$\int \frac{dx}{\sqrt{ax^2+bx+c}} = -\frac{1}{\sqrt{-a}}\arcsin\frac{2ax+b}{\sqrt{b^2-4ac}} \quad (\text{per } a<0,\ 4ac-b^2<0)$$

$$\int \frac{x\,dx}{\sqrt{ax^2+bx+c}} = \frac{\sqrt{ax^2+bx+c}}{a}-\frac{b}{2a}\int\frac{dx}{\sqrt{ax^2+bx+c}}$$

Integrali indefiniti di funzioni trigonometriche

Integrali di funzioni trigonometriche contenenti solo Seno

$$\int \sin cx\, dx = -\frac{1}{c}\cos cx$$

$$\int \sin^n cx\, dx = -\frac{\sin^{n-1} cx \cos cx}{nc} + \frac{n-1}{n}\int \sin^{n-2} cx\, dx \quad (\text{per } n > 0)$$

$$\int x \sin cx\, dx = \frac{\sin cx}{c^2} - \frac{x\cos cx}{c}$$

$$\int x^n \sin cx\, dx = -\frac{x^n}{c}\cos cx + \frac{n}{c}\int x^{n-1}\cos cx\, dx \quad (\text{per } n > 0)$$

$$\int \frac{\sin cx}{x} dx = \sum_{i=0}^{\infty}(-1)^i \frac{(cx)^{2i+1}}{(2i+1)\cdot(2i+1)!}$$

$$\int \frac{\sin cx}{x^n} dx = -\frac{\sin cx}{(n-1)x^{n-1}} + \frac{c}{n-1}\int \frac{\cos cx}{x^{n-1}} dx$$

$$\int \frac{dx}{\sin cx} = \frac{1}{c}\ln\left|\tan\frac{cx}{2}\right|$$

$$\int \frac{dx}{\sin^n cx} = \frac{\cos cx}{c(1-n)\sin^{n-1} cx} + \frac{n-2}{n-1}\int \frac{dx}{\sin^{n-2} cx} \quad (\text{per } n > 1)$$

$$\int \frac{dx}{1 \pm \sin cx} = \frac{1}{c}\tan\left(\frac{cx}{2} \mp \frac{\pi}{4}\right)$$

$$\int \frac{x\, dx}{1 + \sin cx} = \frac{x}{c}\tan\left(\frac{cx}{2} - \frac{\pi}{4}\right) + \frac{2}{c^2}\ln\left|\cos\left(\frac{cx}{2} - \frac{\pi}{4}\right)\right|$$

$$\int \frac{x\, dx}{1 - \sin cx} = \frac{x}{c}\cot\left(\frac{\pi}{4} - \frac{cx}{2}\right) + \frac{2}{c^2}\ln\left|\sin\left(\frac{\pi}{4} - \frac{cx}{2}\right)\right|$$

$$\int \frac{\sin cx\, dx}{1 \pm \sin cx} = \pm x + \frac{1}{c}\tan\left(\frac{pi}{4} \mp \frac{cx}{2}\right)$$

$$\int \sin c_1 x \sin c_2 x\, dx = \frac{\sin(c_1 - c_2)x}{2(c_1 - c_2)} - \frac{\sin(c_1 + c_2)x}{2(c_1 + c_2)} \quad (\text{per } |c_1| \neq |c_2|)$$

Integrali di funzioni trigonometriche contenenti solo Coseno

$$\int \cos cx\, dx = \frac{1}{c}\sin cx$$

$$\int \cos^n cx\, dx = \frac{\cos^{n-1} cx \sin cx}{nc} + \frac{n-1}{n}\int \cos^{n-2} cx\, dx \quad (\text{per } n > 0)$$

$$\int x\cos cx\, dx = \frac{\cos cx}{c^2} + \frac{x\sin cx}{c}$$

$$\int x^n \cos cx\, dx = \frac{x^n \sin cx}{c} - \frac{n}{c}\int x^{n-1}\sin cx\, dx$$

$$\int \frac{\cos cx}{x} dx = \ln|cx| + \sum_{i=1}^{\infty}(-1)^i \frac{(cx)^{2i}}{2i\cdot(2i)!}$$

$$\int \frac{\cos cx}{x^n} dx = -\frac{\cos cx}{(n-1)x^{n-1}} - \frac{c}{n-1}\int \frac{\sin cx}{x^{n-1}} dx \quad (\text{per } n \neq 1)$$

$$\int \frac{dx}{\cos cx} = \frac{1}{c}\ln\left|\tan\left(\frac{cx}{2} + \frac{\pi}{4}\right)\right|$$

$$\int \frac{dx}{\cos^n cx} = \frac{\sin cx}{c(n-1)cos^{n-1} cx} + \frac{n-2}{n-1}\int \frac{dx}{\cos^{n-2} cx} \quad (\text{per } n > 1)$$

$$\int \frac{dx}{1 + \cos cx} = \frac{1}{c}\tan\frac{cx}{2}$$

$$\int \frac{dx}{1 - \cos cx} = -\frac{1}{c}\cot\frac{cx}{2}$$

$$\int \frac{x\, dx}{1 + \cos cx} = \frac{x}{c}\tan cx2 + \frac{2}{c^2}\ln\left|\cos\frac{cx}{2}\right|$$

$$\int \frac{x\,dx}{1-\cos cx} = -\frac{x}{x}\cot cx2 + \frac{2}{c^2}\ln\left|\sin\frac{cx}{2}\right|$$
$$\int \frac{\cos cx\,dx}{1+\cos cx} = x - \frac{1}{c}\tan\frac{cx}{2}$$
$$\int \frac{\cos cx\,dx}{1-\cos cx} = -x - \frac{1}{c}\cot\frac{cx}{2}$$
$$\int \cos c_1 x \cos c_2 x\,dx = \frac{\sin(c_1-c_2)x}{2(c_1-c_2)} + \frac{\sin(c_1+c_2)x}{2(c_1+c_2)} \quad (\text{per } |c_1| \neq |c_2|)$$

Integrali di funzioni trigonometriche contenenti solo Tangente

$$\int \tan cx\,dx = -\frac{1}{c}\ln|\cos cx|$$
$$\int \tan^n cx\,dx = \frac{1}{c(n-1)}\tan^{n-1} cx - \int \tan^{n-2} cx\,dx \quad (\text{per })n \neq 1)$$
$$\int \frac{dx}{\tan cx + 1} = \frac{x}{2} + \frac{1}{2c}\ln|\sin cx + \cos cx|$$
$$\int \frac{dx}{\tan cx - 1} = -\frac{x}{2} + \frac{1}{2c}\ln|\sin cx - \cos cx|$$
$$\int \frac{\tan cx\,dx}{\tan cx + 1} = \frac{x}{2} - \frac{1}{2c}\ln|\sin cx + \cos cx|$$
$$\int \frac{\tan cx\,dx}{\tan cx - 1} = \frac{x}{2} + \frac{1}{2c}\ln|\sin cx - \cos cx|$$

Integrali di funzioni trigonometriche contenenti solo Secante

$$\int \sec cx\,dx = \frac{1}{c}\ln|\sec cx + \tan cx|$$
$$\int \sec^n cx\,dx = \frac{\sec^{n-1} cx \sin cx}{c(n-1)} + \frac{n-2}{n-1}\int \sec^{n-2} cx\,dx \qquad \text{per } n \neq 1,\ c \neq 0$$

Integrali di funzioni trigonometriche contenenti solo Cosecante

$$\int \csc cx\,dx = -\frac{1}{c}\ln|\csc cx + \cot cx|$$
$$\int \csc^n cx\,dx = -\frac{\csc^{n-1} cx \cos cx}{c(n-1)} + \frac{n-2}{n-1}\int \csc^{n-2} cx\,dx \qquad \text{per } n \neq 1,\ c \neq 0$$

Integrali di funzioni trigonometriche contenenti solo Cotangente

$$\int \cot cx\,dx = \frac{1}{c}\ln|\sin cx|$$
$$\int \cot^n cx\,dx = -\frac{1}{c(n-1)}\cot^{n-1} cx - \int \cot^{n-2} cx\,dx \quad (\text{per })n \neq 1)$$
$$\int \frac{dx}{1+\cot cx} = \int \frac{\tan cx\,dx}{\tan cx + 1}$$
$$\int \frac{dx}{1-\cot cx} = \int \frac{\tan cx\,dx}{\tan cx - 1}$$

Integrali di funzioni trigonometriche contenenti solo Seno e Coseno

$$\int \frac{dx}{\cos cx \pm \sin cx} = \frac{1}{c\sqrt{2}}\ln\left|\tan\left(\frac{cx}{2} \pm \frac{\pi}{8}\right)\right|$$
$$\int \frac{dx}{(\cos cx \pm \sin cx)^2} = \frac{1}{2c}\tan\left(cx \mp \frac{\pi}{4}\right)$$

$$\int \frac{\cos cx\;dx}{\cos cx + \sin cx} = \frac{x}{2} + \frac{1}{2c}\ln|\sin cx + \cos cx|$$
$$\int \frac{\cos cx\;dx}{\cos cx - \sin cx} = \frac{x}{2} - \frac{1}{2c}\ln|\sin cx - \cos cx|$$
$$\int \frac{\sin cx\;dx}{\cos cx + \sin cx} = \frac{x}{2} - \frac{1}{2c}\ln|\sin cx + \cos cx|$$
$$\int \frac{\sin cx\;dx}{\cos cx - \sin cx} = -\frac{x}{2} - \frac{1}{2c}\ln|\sin cx - \cos cx|$$
$$\int \frac{\cos cx\;dx}{\sin cx(1+\cos cx)} = -\frac{1}{4c}\tan^2\frac{cx}{2} + \frac{1}{2c}\ln\left|\tan\frac{cx}{2}\right|$$
$$\int \frac{\cos cx\;dx}{\sin cx(1+-\cos cx)} = -\frac{1}{4c}\cot^2\frac{cx}{2} - \frac{1}{2c}\ln\left|\tan\frac{cx}{2}\right|$$
$$\int \frac{\sin cx\;dx}{\cos cx(1+\sin cx)} = \frac{1}{4c}\cot^2\left(\frac{cx}{2}+\frac{\pi}{4}\right) + \frac{1}{2c}\ln\left|\tan\left(\frac{cx}{2}+\frac{\pi}{4}\right)\right|$$
$$\int \frac{\sin cx\;dx}{\cos cx(1-\sin cx)} = \frac{1}{4c}\tan^2\left(\frac{cx}{2}+\frac{\pi}{4}\right) - \frac{1}{2c}\ln\left|\tan\left(\frac{cx}{2}+\frac{\pi}{4}\right)\right|$$
$$\int \sin cx\cos cx\;dx = \frac{1}{2c}\sin^2 cx$$
$$\int \sin c_1x\cos c_2x\;dx = -\frac{\cos(c_1+c_2)x}{2(c_1+c_2)} - \frac{\cos(c_1-c_2)x}{2(c_1-c_2)} \quad (\text{per } |c_1| \neq |c_2|)$$
$$\int \sin^n cx\cos cx\;dx = \frac{1}{c(n+1)}\sin^{n+1} cx \quad (\text{per } n \neq 1)$$
$$\int \sin cx\cos^n cx\;dx = -\frac{1}{c(n+1)}\cos^{n+1} cx \quad (\text{per } n \neq 1)$$
$$\int \sin^n cx\cos^m cx\;dx = -\frac{\sin^{n-1} cx\cos^{m+1} cx}{c(n+m)} + \frac{n-1}{n+m}\int \sin^{n-2} cx\cos^m cx\;dx \quad (\text{per } m,n>0)$$
$$\int \sin^n cx\cos^m cx\;dx = \frac{\sin^{n+1} cx\cos^{m-1} cx}{c(n+m)} + \frac{m-1}{n+m}\int \sin^{n} cx\cos^{m-2} cx\;dx \quad (\text{per } m,n>0)$$
$$\int \frac{dx}{\sin cx\cos cx} = \frac{1}{c}\ln|\tan cx|$$
$$\int \frac{dx}{\sin cx\cos^n cx} = \frac{1}{c(n-1)\cos^{n-1} cx} + \int \frac{dx}{\sin cx\cos^{n-2} cx} \quad (\text{for } n \neq 1)$$
$$\int \frac{dx}{\sin^n cx\cos cx} = -\frac{1}{c(n-1)\sin^{n-1} cx} + \int \frac{dx}{\sin^{n-2} cx\cos cx} \quad (\text{per } n \neq 1)$$
$$\int \frac{\sin cx\;dx}{\cos^n cx} = \frac{1}{c(n-1)\cos^{n-1} cx} \quad (\text{per } n \neq 1)$$
$$\int \frac{\sin^2 cx\;dx}{\cos cx} = -\frac{1}{c}\sin cx + \frac{1}{c}\ln\left|\tan\left(\frac{\pi}{4}+\frac{cx}{2}\right)\right|$$
$$\int \frac{\sin^2 cx\;dx}{\cos^n cx} = \frac{\sin cx}{c(n-1)\cos^{n-1} cx} - \frac{1}{n-1}\int \frac{dx}{\cos^{n-2} cx} \quad (\text{per } n \neq 1)$$
$$\int \frac{\sin^n cx\;dx}{\cos cx} = -\frac{\sin^{n-1} cx}{c(n-1)} + \int \frac{\sin^{n-2} cx\;dx}{\cos cx} \quad (\text{per } n \neq 1)$$
$$\int \frac{\sin^n cx\;dx}{\cos^m cx} = \frac{\sin^{n+1} cx}{c(m-1)\cos^{m-1} cx} - \frac{n-m+2}{m-1}\int \frac{\sin^n cx\;dx}{\cos^{m-2} cx} \quad (\text{per } m \neq 1)$$
$$\int \frac{\sin^n cx\;dx}{\cos^m cx} = -\frac{\sin^{n-1} cx}{c(n-m)\cos^{m-1} cx} + \frac{n-1}{n-m}\int \frac{\sin^{n-2} cx\;dx}{\cos^{m} cx} \quad (\text{per } m \neq n)$$
$$\int \frac{\sin^n cx\;dx}{\cos^m cx} = \frac{\sin^{n-1} cx}{c(m-1)\cos^{m-1} cx} - \frac{n-1}{n-1}\int \frac{\sin^{n-1} cx\;dx}{\cos^{m-2} cx} \quad (\text{per } m \neq 1)$$
$$\int \frac{\cos cx\;dx}{\sin^n cx} = -\frac{1}{c(n-1)\sin^{n-1} cx} \quad (\text{per } n \neq 1)$$
$$\int \frac{\cos^2 cx\;dx}{\sin cx} = \frac{1}{c}\left(\cos cx + \ln\left|\tan\frac{cx}{2}\right|\right)$$
$$\int \frac{\cos^2 cx\;dx}{\sin^n cx} = -\frac{1}{n-1}\left(\frac{\cos cx}{c\sin^{n-1} cx)} + \int \frac{dx}{\sin^{n-2} cx}\right) \quad (\text{per } n \neq 1)$$

$$\int \frac{\cos^n cx\, dx}{\sin^m cx} = -\frac{\cos^{n+1} cx}{c(m-1)\sin^{m-1} cx} - \frac{n-m-2}{m-1}\int \frac{cos^n cx\, dx}{\sin^{m-2} cx} \quad \text{(per } m \neq 1)$$
$$\int \frac{\cos^n cx\, dx}{\sin^m cx} = \frac{\cos^{n-1} cx}{c(n-m)\sin^{m-1} cx} + \frac{n-1}{n-m}\int \frac{cos^{n-2} cx\, dx}{\sin^m cx} \quad \text{(per } m \neq n)$$
$$\int \frac{\cos^n cx\, dx}{\sin^m cx} = -\frac{\cos^{n-1} cx}{c(m-1)\sin^{m-1} cx} - \frac{n-1}{m-1}\int \frac{cos^{n-2} cx\, dx}{\sin^{m-2} cx} \quad \text{(per } m \neq 1)$$

Integrali di funzioni trigonometriche contenenti solo Seni, Coseni e Tangenti

$$\int \sin cx \tan cx\, dx = \frac{1}{c}(\ln|\sec cx + \tan cx| - \sin cx)$$
$$\int \frac{\tan^n cx\, dx}{\sin^2 cx} = \frac{1}{c(n-1)}\tan^{n-1}(cx) \quad \text{(per } n \neq 1)$$
$$\int \frac{\tan^n cx\, dx}{\cos^2 cx} = \frac{1}{c(n+1)}\tan^{n+1} cx \quad \text{(per } n \neq -1)$$
$$\int \frac{\cot^n cx\, dx}{\sin^2 cx} = \frac{1}{c(n+1)}\cot^{n+1} cx \quad \text{(per } n \neq -1)$$
$$\int \frac{\cot^n cx\, dx}{\cos^2 cx} = \frac{1}{c(1-n)}\tan^{1-n} cx \quad \text{(per } n \neq 1)$$
$$\int \frac{\tan^m(cx)}{\cot^n(cx)}\, dx = \frac{1}{c(m+n-1)}\tan^{m+n-1}(cx) - \int \frac{\tan^{m-2}(cx)}{\cot^n(cx)}\, dx \quad \text{(per } m+n \neq 1)$$

Integrali indefiniti di funzioni iperboliche

$$\int \sinh cx\,dx = \frac{1}{c}\cosh cx$$

$$\int \cosh cx\,dx = \frac{1}{c}\sinh cx$$

$$\int \sinh^2 cx\,dx = \frac{1}{4c}\sinh 2cx - \frac{x}{2}$$

$$\int \cosh^2 cx\,dx = \frac{1}{4c}\sinh 2cx + \frac{x}{2}$$

$$\int \sinh^n cx\,dx = \frac{1}{cn}\sinh^{n-1} cx\cosh cx - \frac{n-1}{n}\int \sinh^{n-2} cx\,dx \quad (\text{per } n > 0)$$

$$\int \sinh^n cx\,dx = \frac{1}{c(n+1)}\sinh^{n+1} cx\cosh cx - \frac{n+2}{n+1}\int \sinh^{n+2} cx\,dx \quad (\text{per } n < 0,\ n \neq -1)$$

$$\int \cosh^n cx\,dx = \frac{1}{cn}\sinh cx\cosh^{n-1} cx + \frac{n-1}{n}\int \cosh^{n-2} cx\,dx \quad (\text{per } n > 0)$$

$$\int \cosh^n cx\,dx = -\frac{1}{c(n+1)}\sinh cx\cosh^{n+1} cx - \frac{n+2}{n+1}\int \cosh^{n+2} cx\,dx \quad (\text{per } n < 0,\ n \neq -1)$$

$$\int \frac{dx}{\sinh cx} = \frac{1}{c}\ln\left|\tanh\frac{cx}{2}\right|$$

$$\int \frac{dx}{\sinh cx} = \frac{1}{c}\ln\left|\frac{\cosh cx - 1}{\sinh cx}\right|$$

$$\int \frac{dx}{\sinh cx} = \frac{1}{c}\ln\left|\frac{\sinh cx}{\cosh cx + 1}\right|$$

$$\int \frac{dx}{\sinh cx} = \frac{1}{c}\ln\left|\frac{\cosh cx - 1}{\cosh cx + 1}\right|$$

$$\int \frac{dx}{\cosh cx} = \frac{2}{c}\arctan e^{cx}$$

$$\int \frac{dx}{\sinh^n cx} = \frac{\cosh cx}{c(n-1)\sinh^{n-1} cx} - \frac{n-2}{n-1}\int \frac{dx}{\sinh^{n-2} cx} \quad (\text{per } n \neq 1)$$

$$\int \frac{dx}{\cosh^n cx} = \frac{\sinh cx}{c(n-1)\cosh^{n-1} cx} + \frac{n-2}{n-1}\int \frac{dx}{\cosh^{n-2} cx} \quad (\text{per } n \neq 1)$$

$$\int \frac{\cosh^n cx}{\sinh^m cx}dx = \frac{\cosh^{n-1} cx}{c(n-m)\sinh^{m-1} cx} + \frac{n-1}{n-m}\int \frac{\cosh^{n-2} cx}{\sinh^m cx}dx \quad (\text{per } m \neq n)$$

$$\int \frac{\cosh^n cx}{\sinh^m cx}dx = -\frac{\cosh^{n+1} cx}{c(m-1)\sinh^{m-1} cx} + \frac{n-m+2}{m-1}\int \frac{\cosh^n cx}{\sinh^{m-2} cx}dx \quad (\text{per } m \neq 1)$$

$$\int \frac{\cosh^n cx}{\sinh^m cx}dx = -\frac{\cosh^{n-1} cx}{c(m-1)\sinh^{m-1} cx} + \frac{n-1}{m-1}\int \frac{\cosh^{n-2} cx}{\sinh^{m-2} cx}dx \quad (\text{per } m \neq 1)$$

$$\int \frac{\sinh^m cx}{\cosh^n cx}dx = \frac{\sinh^{m-1} cx}{c(m-n)\cosh^{n-1} cx} + \frac{m-1}{m-n}\int \frac{\sinh^{m-2} cx}{\cosh^n cx}dx \quad (\text{per } m \neq n)$$

$$\int \frac{\sinh^m cx}{\cosh^n cx}dx = \frac{\sinh^{m+1} cx}{c(n-1)\cosh^{n-1} cx} + \frac{m-n+2}{n-1}\int \frac{\sinh^m cx}{\cosh^{n-2} cx}dx \quad (\text{per } n \neq 1)$$

$$\int \frac{\sinh^m cx}{\cosh^n cx}dx = -\frac{\sinh^{m-1} cx}{c(n-1)\cosh^{n-1} cx} + \frac{m-1}{n-1}\int \frac{\sinh^{m-2} cx}{\cosh^{n-2} cx}dx \quad (\text{per } n \neq 1)$$

$$\int x\sinh cx\,dx = \frac{1}{c}x\cosh cx - \frac{1}{c^2}\sinh cx$$

$$\int x\cosh cx\,dx = \frac{1}{c}x\sinh cx - \frac{1}{c^2}\cosh cx$$

$$\int \tanh cx\,dx = \frac{1}{c}\ln|\cosh cx|$$

$$\int \coth cx\,dx = \frac{1}{c}\ln|\sinh cx|$$

$$\int \tanh^n cx\,dx = -\frac{1}{c(n-1)}\tanh^{n-1} cx + \int \tanh^{n-2} cx\,dx \quad (\text{per } n \neq 1)$$

$$\int \coth^n cx\,dx = -\frac{1}{c(n-1)}\coth^{n-1} cx + \int \coth^{n-2} cx\,dx \quad (\text{per } n \neq 1)$$
$$\int \sinh bx \sinh cx\,dx = \frac{1}{b^2-c^2}(b\sinh cx\cosh bx - c\cosh cx\sinh bx) \quad (\text{per } b^2 \neq c^2)$$
$$\int \cosh bx \cosh cx\,dx = \frac{1}{b^2-c^2}(b\sinh bx\cosh cx - c\sinh cx\cosh bx) \quad (\text{per } b^2 \neq c^2)$$
$$\int \cosh bx \sinh cx\,dx = \frac{1}{b^2-c^2}(b\sinh bx\sinh cx - c\cosh bx\cosh cx) \quad (\text{per } b^2 \neq c^2)$$
$$\int \sinh(ax+b)\sin(cx+d)\,dx = \frac{a}{a^2+c^2}\cosh(ax+b)\sin(cx+d) - \frac{c}{a^2+c^2}\sinh(ax+b)\cos(cx+d)$$
$$\int \sinh(ax+b)\cos(cx+d)\,dx = \frac{a}{a^2+c^2}\cosh(ax+b)\cos(cx+d) + \frac{c}{a^2+c^2}\sinh(ax+b)\sin(cx+d)$$
$$\int \cosh(ax+b)\sin(cx+d)\,dx = \frac{a}{a^2+c^2}\sinh(ax+b)\sin(cx+d) - \frac{c}{a^2+c^2}\cosh(ax+b)\cos(cx+d)$$
$$\int \cosh(ax+b)\cos(cx+d)\,dx = \frac{a}{a^2+c^2}\sinh(ax+b)\cos(cx+d) + \frac{c}{a^2+c^2}\cosh(ax+b)\sin(cx+d)$$

Integrali indefiniti di funzioni esponenziali

$$\int e^{cx}\;dx = \frac{1}{c}e^{cx}$$
$$\int a^{cx}\;dx = \frac{1}{c\ln a}a^{cx} \quad (\text{per } a > 0,\; a \neq 1)$$
$$\int xe^{cx}\;dx = \frac{e^{cx}}{c^2}(cx-1)$$
$$\int x^2e^{cx}\;dx = e^{cx}\left(\frac{x^2}{c} - \frac{2x}{c^2} + \frac{2}{c^3}\right)$$
$$\int x^n e^{cx}\;dx = \frac{1}{c}x^n e^{cx} - \frac{n}{c}\int x^{n-1}e^{cx}dx$$
$$\int \frac{e^{cx}\;dx}{x} = \ln|x| + \sum_{i=1}^{\infty}\frac{(cx)^i}{i\cdot i!}$$
$$\int \frac{e^{cx}\;dx}{x^n} = \frac{1}{n-1}\left(-\frac{e^{cx}}{x^{n-1}} + c\int \frac{e^{cx}dx}{x^{n-1}}\right) \quad (\text{per } n \neq 1)$$
$$\int e^{cx}\ln x\;dx = \frac{1}{c}\left(e^{cx}\ln|x| - \int \frac{e^{cx}dx}{x}\right)$$
$$\int e^{cx}\sin bx\;dx = \frac{e^{cx}}{c^2+b^2}(c\sin bx - b\cos bx)$$
$$\int e^{cx}\cos bx\;dx = \frac{e^{cx}}{c^2+b^2}(c\cos bx + b\sin bx)$$
$$\int e^{cx}\sin^n x\;dx = \frac{e^{cx}\sin^{n-1}x}{c^2+n^2}(c\sin x - n\cos x) + \frac{n(n-1)}{c^2+n^2}\int e^{cx}\sin^{n-2}x\;dx$$
$$\int e^{cx}\cos^n x\;dx = \frac{e^{cx}\cos^{n-1}x}{c^2+n^2}(c\cos x + n\sin x) + \frac{n(n-1)}{c^2+n^2}\int e^{cx}\cos^{n-2}x\;dx$$
$$\int \frac{1}{\sigma\sqrt{2\pi}}\,e^{-(x-\mu)^2/2\sigma^2} = \frac{1}{2\sigma}(1 + \operatorname{erf}\frac{x-\mu}{\sigma\sqrt{(2)}})$$

Integrali indefiniti di funzioni d'arco

In questa pagina si assume che c denoti una costante diversa da 0. $\int \arcsin\frac{x}{c}\,dx = x\arcsin\frac{x}{c}+\sqrt{c^2-x^2}$

$$\int x\arcsin\frac{x}{c}\,dx = \left(\frac{x^2}{2}-\frac{c^2}{4}\right)\arcsin\frac{x}{c}+\frac{x}{4}\sqrt{c^2-x^2}$$

$$\int x^2\arcsin\frac{x}{c}\,dx = \frac{x^3}{3}\arcsin\frac{x}{c}+\frac{x^2+2c^2}{9}\sqrt{c^2-x^2}$$

$$\int \arccos\frac{x}{c}\,dx = x\arccos\frac{x}{c}-\sqrt{c^2-x^2}$$

$$\int x\arccos\frac{x}{c}\,dx = \left(\frac{x^2}{2}-\frac{c^2}{4}\right)\arccos\frac{x}{c}-\frac{x}{4}\sqrt{c^2-x^2}$$

$$\int x^2\arccos\frac{x}{c}\,dx = \frac{x^3}{3}\arccos\frac{x}{c}-\frac{x^2+2c^2}{9}\sqrt{c^2-x^2}$$

$$\int \arctan\frac{x}{c}\,dx = x\arctan\frac{x}{c}-\frac{c}{2}\ln(c^2+x^2)$$

$$\int x\arctan\frac{x}{c}\,dx = \frac{c^2+x^2}{2}\arctan\frac{x}{c}-\frac{cx}{2}$$

$$\int x^2\arctan\frac{x}{c}\,dx = \frac{x^3}{3}\arctan\frac{x}{c}-\frac{cx^2}{6}+\frac{c^3}{6}\ln c^2+x^2$$

$$\int x^n\arctan\frac{x}{c}\,dx = \frac{x^{n+1}}{n+1}\arctan\frac{x}{c}-\frac{c}{n+1}\int\frac{x^{n+1}dx}{c^2+x^2}\quad(\text{per } n\neq 1)$$

$$\int \mathrm{arcsec}\frac{x}{c}\,dx = x\mathrm{arcsec}\frac{x}{c}+\frac{x}{c|x|}\ln|x\pm\sqrt{x^2-1}|$$

$$\int \mathrm{arccot}\frac{x}{c}\,dx = x\,\mathrm{arccot}\frac{x}{c}+\frac{c}{2}\ln(c^2+x^2)$$

$$\int x\,\mathrm{arccot}\frac{x}{c}\,dx = \frac{c^2+x^2}{2}\mathrm{arccot}\frac{x}{c}+\frac{cx}{2}$$

$$\int x^2\,\mathrm{arccot}\frac{x}{c}\,dx = \frac{x^3}{3}\mathrm{arccot}\frac{x}{c}+\frac{cx^2}{6}-\frac{c^3}{6}\ln(c^2+x^2)$$

$$\int x^n\,\mathrm{arccot}\frac{x}{c}\,dx = \frac{x^{n+1}}{n+1}\mathrm{arccot}\frac{x}{c}+\frac{c}{n+1}\int\frac{x^{n+1}dx}{c^2+x^2}\quad(\text{per } n\neq 1)$$

Derivate

Regole di derivazione

$D[\alpha f(x) \pm \beta g(x)] = \alpha f'(x) \pm \beta g'(x)$ $\quad$ $D[f(x)g(x)] = D[f(x)] \cdot g(x) + f(x) \cdot D[g(x)]$

$D\dfrac{f(x)}{g(x)} = \dfrac{f'(x)g(x) - g'(x)f(x)}{g(x)^2}$ $\quad$ $D\dfrac{1}{f(x)} = -\dfrac{f'(x)}{f(x)^2}$

$D\left(f^{-1}(y)\right) = \dfrac{1}{f'(x)} = \dfrac{1}{f'(f^{-1}(y))}$ $\quad$ $D[f(g(x))] = f'(g(x)) \cdot g'(x)$

Derivate fondamentali

$D(a) = 0$ $\quad$ $D(ax) = a$

$D(x^2) = 2x$ $\quad$ $D(x^n) = nx^{n-1}$

$D(\sqrt[n]{x}) = \dfrac{1}{n\sqrt[n]{x^{n-1}}}$ se $x > 0$ $\quad$ $D(x^\alpha) = \alpha x^{\alpha-1}, \alpha \in \mathbb{R}$

$D(|x|) = \begin{cases} 1 & \text{se } x > 0 \\ -1 & \text{se } x < 0 \\ \text{non derivabile} & \text{se } x = 0 \end{cases}$ $\quad$ $D(e^x) = e^x$

$D(a^x) = a^x \ln a$

$D(\ln x) = \dfrac{1}{x}$ $\quad$ $D(\log_b x) = \dfrac{1}{x \ln b} = \dfrac{\log_b e}{x}$

$D(\sin x) = \cos x$ $\quad$ $D(\cos x) = -\sin x$

$D(\tan x) = 1 + \tan^2 x = \dfrac{1}{\cos^2 x}$ $\quad$ $D(\cot x) = -1 - \cot^2 x = -\dfrac{1}{\sin^2 x}$

$D(\sec x) = \tan x \sec x$ $\quad$ $D(\csc x) = -\cot x \csc x$

$D(\arcsin x) = \dfrac{1}{\sqrt{1-x^2}}$ $\quad$ $D(\arccos x) = \dfrac{1}{-\sqrt{1-x^2}}$

$D(\arctan x) = \dfrac{1}{1+x^2}$ $\quad$ $D(\text{arccot}x) = \dfrac{-1}{1+x^2}$

$D(\text{arcsec}x) = \dfrac{1}{|x|\sqrt{x^2-1}}$ $\quad$ $D(\text{arccsc}x) = \dfrac{-1}{|x|\sqrt{x^2-1}}$

$D(\sinh x) = \cosh x$ $\quad$ $D(\cosh x) = \sinh x$

$D(\tanh x) = \dfrac{1}{\cosh^2 x}$ $\quad$ $D(\text{coth}x) = -\text{csch}^2 x$

$D(\text{sech}x) = -\tanh x \text{sech}x$ $\quad$ $D(\text{csch}x) = -\text{coth}x\text{csch}x$

$D(\text{arcsinh}x) = \dfrac{1}{\sqrt{x^2+1}}$ $\quad$ $D(\text{arccosh}x) = \dfrac{-1}{\sqrt{x^2-1}}$

$D(\text{arctanh}x) = \dfrac{1}{1-x^2}$ $\quad$ $D(\text{arccoth}x) = \dfrac{-1}{1-x^2}$

$D(\text{arcsech}x) = \dfrac{1}{x\sqrt{1-x^2}}$ $\quad$ $D(\text{arccsch}x) = \dfrac{-1}{|x|\sqrt{1+x^2}}$

$D([f(x)]^n) = nf(x)^{n-1}f'(x)$ $\quad$ $D(\sqrt[n]{f(x)}) = \dfrac{f'(x)}{n\sqrt[n]{f(x)^{n-1}}}$

$D(lnf(x)) = \dfrac{f'(x)}{f(x)}$ $\quad$ $D(e^{f(x)}) = e^{f(x)}f'(x)$

$D(sinf(x)) = f'(x)cosf(x)$ $\quad$ $D(cosf(x)) = -f'(x)sinf(x)$

$D(\arctan f(x)) = \dfrac{f'(x)}{1+[f(x)]^2}$ $\quad$ $D(f(x)^{g(x)}) = f(x)^{g(x)}\left\{g'(x)lnf(x) + g(x)\dfrac{f'(x)}{f(x)}\right\}$

Sviluppi in serie di Taylor di funzioni di uso comune

I seguenti sono alcuni importanti sviluppi in serie di Taylor in un intorno di 0. Tutti questi sviluppi sono validi anche per argomenti x complessi.

$$\mathrm{e}^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} \approx 1 + x + x^2/2 + ...$$

$$\ln(1+x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} x^n \approx x - x^2/2 + x^3/3 + ... \quad \text{per } |x| < 1$$

$$\frac{x^m}{1-x} = \sum_{n=m}^{\infty} x^n \quad \text{per } |x| < 1$$

$$(1+x)^\alpha = \sum_{n=0}^{\infty} \binom{\alpha}{n} x^n \approx 1 + \alpha x + \alpha(\alpha-1)x^2/2 \quad \text{per ogni } |x| < 1$$

$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1} \approx x - x^3/6 + x^5/120 \quad \text{per ogni } x$$

$$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n} \approx 1 - x^2/2 + x^4/24 \quad \text{per ogni } x$$

$$\tan x = \sum_{n=1}^{\infty} \frac{B_{2n}(-4)^n(1-4^n)}{(2n)!} x^{2n-1} \approx x + x^3/3 + 2/15x^5 + ... \quad \text{per } |x| < \frac{\pi}{2}$$

$$\sec x = \sum_{n=0}^{\infty} \frac{(-1)^n E_{2n}}{(2n)!} x^{2n} \approx 1 + x^2/2 + 5/24x^4 + ... \quad \text{per } |x| < \frac{\pi}{2}$$

$$\arcsin x = \sum_{n=0}^{\infty} \frac{(2n)!}{4^n(n!)^2(2n+1)} x^{2n+1} \approx x + x^3/6 + 3/40x^5 + ... \quad \text{per } |x| < 1$$

$$\arctan x = \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} x^{2n+1} \approx x - x^3/3 + x^5/5 + ... \quad \text{per } |x| < 1$$

$$\sinh(x) = \sum_{n=0}^{\infty} \frac{1}{(2n+1)!} x^{2n+1} \approx x + x^3/6 + x^5/120 + ... \quad \text{per ogni } x$$

$$\cosh(x) = \sum_{n=0}^{\infty} \frac{1}{(2n)!} x^{2n} \approx 1 + x^2/2 + x^4/24 + ... \quad \text{per ogni } x$$

$$\tanh(x) = \sum_{n=1}^{\infty} \frac{B_{2n}4^n(4^n-1)}{(2n)!} x^{2n-1} \approx x - x^3/3 + 2/15x^5 + ... \quad \text{per } |x| < \frac{\pi}{2}$$

$$\operatorname{arcsinh}(x) = \sum_{n=0}^{\infty} \frac{(-1)^n(2n)!}{4^n(n!)^2(2n+1)} x^{2n+1} \approx x - x^3/6 + 3/40x^5 + ... \quad \text{per } |x| < 1$$

$$\operatorname{arctanh}(x) = \sum_{n=0}^{\infty} \frac{1}{2n+1} x^{2n+1} \approx x + x^3/3 + x^5/5 + ... \quad \text{per } |x| < 1$$

I numeri B_k che compaiono negli sviluppi di $\tan(x)$ e $\tanh(x)$ sono i numeri di Bernoulli. Lo sviluppo binomiale si serve dei coefficienti binomiali. Gli E_k nello sviluppo della $\sec(x)$ sono i numeri di Eulero.

$B_k = \{1, -1/2, 1/6, 0, -1/30, 0, 1/42, 0, -1/30, 0, 5/66, 0, -691/2730, 0, 7/6, ...\}$

$E_k = \{1, -0, -1, 0, 5, 0, -61, 0, 1385, 0, -50521, 0, 2702765, 0, -199360981, ...\}$

Funzioni Notevoli

Funzione Gamma

La funzione Gamma, nota anche come *funzione gamma di Eulero* é una funzione meromorfa, funzione continua sui numeri reali positivi, che estende il concetto di fattoriale ai numeri complessi, nel senso che per ogni numero intero non negativo n si ha

$$\Gamma(n+1) = n!$$

Def: Se la parte reale del numero complesso z é positiva, allora l'integrale

$$\Gamma(z) = \int_0^{\infty} t^{z-1}\, e^{-t}\, dt$$

converge assolutamente. Comunque, usando la continuazione analitica, si puó vedere che la Γ converge anche per z con parte reale non positiva, purché non intera. Usando l'integrazione per parti, si puó dimostrare che:

$$\Gamma(z+1) = z\Gamma(z)$$

Def Alternativa: Le seguenti definizioni alternative per la funzione Gamma sono dovute a Carl Friedrich Gauss e Karl Weierstrass rispettivamente, sono valide per tutti i complessi z a parte reale non positiva:

$$\Gamma(z) = \lim_{n\to\infty} \frac{n!\, n^z}{z\,(z+1)\cdots(z+n)}$$

$$\Gamma(z) = \frac{e^{-\gamma z}}{z} \prod_{n=1}^{\infty} \left(1 + \frac{z}{n}\right)^{-1} e^{z/n}$$

dove γ é la costante di Eulero-Mascheroni.
Proprietá
Altre importanti proprietá della funzione Gamma sono la formula di riflessione di Eulero

$$\Gamma(1-z)\,\Gamma(z) = \frac{\pi}{\sin \pi z}$$

e quella di duplicazione

$$\Gamma(z)\,\Gamma\left(z+\frac{1}{2}\right) = 2^{1-2z}\,\sqrt{\pi}\,\Gamma(2z)$$

La formula di duplicazione é un caso particolare della formula di moltiplicazione:

$$\Gamma(z)\,\Gamma\left(z+\frac{1}{m}\right)\,\Gamma\left(z+\frac{2}{m}\right)\cdots\Gamma\left(z+\frac{m-1}{m}\right) = (2\pi)^{(m-1)/2}\,m^{1/2-mz}\,\Gamma(mz).$$

Le derivate della Funzione Gamma sono descritte in termine di gamma ed altre funzioni, per esempio:

$$\Gamma'(z) = \Gamma(z)\left[-\gamma - \frac{1}{x} + \sum_{k=0}^{\infty}\left(\frac{1}{k} - \frac{1}{x+k}\right)\right]$$

Il teorema di Bohr-Mollerup (Artin per l'Abbo) afferma che tra tutte le funzioni che estendono la funzione fattoriale, solo Gamma é tale che $\ln(\Gamma(z))$ é convessa.

Funzione Beta

La funzione Beta di Leonhard Euler, detta anche Integrale di Eulero del primo tipo, é data dall'Tavola degli integrali definiti:

$$\beta(x,y) = \int_0^1 t^{x-1}(1-t)^{y-1} dt$$

dove sia x che y hanno parte reale positiva e non nulla (se lo fossero, l'integrale non convergerebbe a un numero finito). Questa funzione fu studiata per primo da Eulero e da Adrien-Marie Legendre, ma fu Jacques Binet a battezzarla con il suo nome attuale. é una funzione simmetrica, cioé il suo valore non cambia scambiando x e y:

$$\beta\,(x, y) = \beta\,(y, x)$$

Inoltre valgono anche le due seguenti identitá: $\beta(1, 1) = 1$ e $\beta(1/2, 1/2) = \pi$;
La funzione Beta si puó scrivere in molti modi, di cui i piú comuni sono i seguenti:

$$\beta(x, y) = \frac{\Gamma(x)\,\Gamma(y)}{\Gamma(x + y)}$$

$$\beta(x, y) = 2\int_0^{\pi/2} \sin^{2x-1}\theta\cos^{2y-1}\theta\,d\theta, \qquad \Re(x) > 0,\ \Re(y) > 0$$

$$\beta(x, y) = \int_0^{\infty} \frac{t^{x-1}}{(1 + t)^{x+y}}\,dt, \qquad \Re(x) > 0,\ \Re(y) > 0$$

$$\beta(x, y) = \frac{1}{y}\sum_{n=0}^{\infty}(-1)^n\,\frac{(y)_{n+1}}{n!(x + n)}$$

Derivata: La derivata della funzione beta puó essere scritta sfruttando, di nuovo, la funzione gamma:

$$\frac{\partial}{\partial x}\beta(x, y) = \beta(x, y)\left(\frac{\Gamma'(x)}{\Gamma(x)} - \frac{\Gamma'(x + y)}{\Gamma(x + y)}\right) = \beta(x, y)(\psi(x) - \psi(x + y))$$

dove $\psi(x)$ é la funzione digamma.

Approssimazione Fattoriale e Stirling

L'approssimazione di Stirling o formula di Stirling é un'approssimazione per fattoriali grandi. Deve il suo nome al matematico scozzese James Stirling (1692-1770). La formulazione corretta é:

$$\lim_{n\to+\infty} \frac{\sqrt{2\pi n}\,\left(\frac{n}{e}\right)^n}{n!} = 1$$

che viene scritto spesso come

$$n! \sim \sqrt{2\pi n}\,\left(\frac{n}{e}\right)^n$$

Per valori elevati di n il secondo membro della formula fornisce una buona approssimazione di n che si puó calcolare rapidamente e facilmente.

Utili Formule Trigonometriche

Simmetrie

$$\begin{array}{lll}
\sin(-x) = -\sin x & \cos(-x) = \cos x & \tan(-x) = -\tan x \\
\sin\left(x + \frac{\pi}{2}\right) = \cos x & \cos\left(x + \frac{\pi}{2}\right) = -\sin x & \tan\left(x + \frac{\pi}{2}\right) = -\frac{1}{\tan x} \\
\sin\left(x + \pi\right) = -\sin x & \cos\left(x + \pi\right) = -\cos x & \tan\left(x + \pi\right) = \tan x \\
\sin\left(\frac{\pi}{2} - x\right) = \cos x & \cos\left(\frac{\pi}{2} - x\right) = \sin x & \tan\left(\frac{\pi}{2} - x\right) = \frac{1}{\tan x}
\end{array}$$

Addizione

$$\cos(x \pm y) = \cos(x)\cos(y) \mp \sin(x)\sin(y) \qquad \sin(x \pm y) = \sin(x)\cos(y) \pm \cos(x)\sin(y)$$

$$\tan(x \pm y) = \frac{\tan(x) \pm \tan(y)}{1 \mp \tan(x)\tan(y)} \qquad \cot(x + y) = \frac{\cot(x)\cdot\cot(y) - 1}{\cot(x) + \cot(y)}$$

$$\cot(x - y) = \frac{\cot(x)\cdot\cot(y) + 1}{\cot(y) - \cot(x)} \qquad \mathrm{cis}(x + y) = \mathrm{cis}(x)\,\mathrm{cis}(y)$$

$$\mathrm{cis}(x - y) = \frac{\mathrm{cis}(x)}{\mathrm{cis(y)}}$$

dove $\mathrm{cis}(x) := e^{ix} = \cos(x) + i\sin(x)$.

Duplicazione

$$\sin(2x) = 2\sin(x)\cos(x) \qquad \cos(2x) = \cos^2(x) - \sin^2(x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x)$$

$$\tan(2x) = \frac{2\tan(x)}{1 - \tan^2(x)} \qquad \cot(2x) = \frac{\cot^2(x) - 1}{2\cot(x)}$$

$$\cos^2(x) = \frac{1 + \cos(2x)}{2} \qquad \sin^2(x) = \frac{1 - \cos(2x)}{2}$$

Cambio di variabile parametrico

Posto $\quad t := \tan\left(\dfrac{x}{2}\right)$,

seguono le cosidette formule parametriche:

$$\sin(x) = \frac{2t}{1 + t^2} \qquad \cos(x) = \frac{1 - t^2}{1 + t^2}$$

$$\tan(x) = \frac{2t}{1 - t^2} \qquad dx = \frac{2dt}{1 + t^2}$$

$$e^{ix} = \frac{1 + it}{1 - it}.$$

Prostaferesi e affini

$$\cos(x)\cos(y) = \frac{\cos(x + y) + \cos(x - y)}{2} \qquad \sin(x)\sin(y) = \frac{\cos(x - y) - \cos(x + y)}{2}$$

$$\sin(x)\cos(y) = \frac{\sin(x + y) + \sin(x - y)}{2} \qquad \sin(x) + \sin(y) = 2\sin\left(\frac{x + y}{2}\right)\cos\left(\frac{x - y}{2}\right)$$

$$\cos(x) + \cos(y) = 2\cos\left(\frac{x + y}{2}\right)\cos\left(\frac{x - y}{2}\right)$$

Inverse

$$\arctan(x) + \arctan(1/x) = \begin{cases} \pi/2, & \text{se } x > 0 \\ -\pi/2, & \text{se } x < 0 \end{cases}. \qquad \arctan(x) + \arctan(y) = \arctan\left(\frac{x + y}{1 - xy}\right)$$

$$\sin^2(\arccos(x)) = 1 - x^2\text{ , per } -1 <= x <= 1 \qquad \cos^2(\arcsin(x)) = 1 - x^2\text{ , per } -1 <= x <= 1$$

$$\sin^2(\arctan(x)) = \frac{x^2}{1 + x^2} \qquad \cos^2(\arctan(x)) = \frac{1}{1 + x^2}$$

Formule di trigonometria iperbolica o come cavolo si chiama

Definizioni

$\cosh(x) = \dfrac{e^x + e^{-x}}{2}$

$\sinh(x) = \dfrac{e^x - e^{-x}}{2}$

$\tanh(x) = \dfrac{\sinh x}{\cosh x} = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$

Utili Identitá

$\cosh^2(x) - \sinh^2(x) = 1$

$\cosh(x \pm y) = \cosh(x)\cosh(y) \pm \sinh(x)\sinh(y)$

$\sinh(x \pm y) = \sinh(x)\cosh(y) \pm \cosh(x)\sinh(y)$

$\tanh(x \pm y) = \dfrac{\tanh(x) \pm \tanh(y)}{1 \pm \tanh(x)\tanh(y)}$

$\sinh(x) \pm \sinh(y) = 2\sinh\left(\dfrac{x+y}{2}\right)\cos\left(\dfrac{x-y}{2}\right)$

$\cosh(x) - \cosh(y) = 2\sinh\left(\dfrac{x-y}{2}\right)\sinh\left(\dfrac{x+y}{2}\right)$

$\tanh(x) - \tanh(y) = \dfrac{\sinh(x-y)}{\cosh(x)\cosh(y)}$

Formule Inverse

$\text{arcsinh}(x) = \log\left(x + \sqrt{x^2+1}\right)$

$\text{arccosh}(x) = \log\left(x + \sqrt{x^2-1}\right)$

$\text{arctanh}(x) = \dfrac{1}{2}\log\dfrac{1+x}{1-x}$