LIMITI NOTEVOLI

TRIGONOMETRICHE

$\lim_{x\to0}\frac{\operatorname{sen}x}{x}=1$	$\lim_{x\to0}\frac{\operatorname{sen}x^n}{x}=1$	$\lim_{x\to\infty}\frac{\operatorname{sen}x}{x}=0$
$\lim_{x\to0}\frac{\operatorname{sen}\alpha x}{x}=\alpha$	$\lim_{x\to0}\frac{\operatorname{sen}\alpha x}{\operatorname{sen}\beta x}=\frac{\alpha}{\beta}$	$\nexists\lim_{x\to\infty}\operatorname{sen}x,\ \lim_{x\to0}\operatorname{sen}\frac{1}{x}$
$\lim_{x\to0}\frac{1-\cos x}{x}=0$	$\lim_{x\to0}\frac{1-\cos x}{x^2}=\frac{1}{2}$	$\nexists\lim_{x\to\infty}\cos x,\ \lim_{x\to0}\cos\frac{1}{x}$
$\lim_{x\to0}\frac{\operatorname{tg}x}{x}=1$	$\lim_{x\to0}\frac{\operatorname{tg}\alpha x}{x}=\alpha$	$\lim_{x\to0}\frac{\operatorname{tg}\alpha x}{\operatorname{tg}\beta x}=\frac{\alpha}{\beta}$

ESPONENZIALE - LOGARITMO – POTENZA

$\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x=e$ $\quad\lim_{x\to0}(1+x)^{1/x}=e$ $\quad\lim_{x\to\infty}\left(1-\frac{1}{x}\right)^x=\frac{1}{e}$

$\lim_{x\to\infty}\left(1+\frac{\alpha}{x}\right)^x=e^\alpha$ $\quad\lim_{x\to\infty}\left(1+\frac{\alpha}{x}\right)^{mx}=e^{m\alpha}$ $\quad\lim_{x\to+\infty}\frac{x^\alpha}{e^x}=0$

$\lim_{x\to+\infty}\frac{e^x}{x^\alpha}=+\infty$ $\quad\lim_{x\to\infty}\frac{x^n}{a^x}=0\ \ (a>1)$ $\quad\lim_{x\to c}\frac{x^n-c^n}{x-c}=n\,c^{n-1}$

$\lim_{x\to0}\frac{a^x-1}{x}=\ln a\ _{(a>0)}$ $\quad\lim_{x\to0}\frac{e^x-1}{x}=1$ $\quad\lim_{x\to0}\frac{(1+x)^\alpha-1}{\alpha x}=1$

$\lim_{x\to0}x^x=1$ $\quad\lim_{x\to0^+}x^\alpha\ln x=0$ $\quad\lim_{x\to+\infty}x^\alpha\ln x=+\infty$

$\lim_{x\to\infty}x^{1/x}=1$ $\quad\lim_{x\to+\infty}\frac{\ln x}{x^\alpha}=0$ $\quad\lim_{x\to0}\frac{\ln(1+x)}{x}=1$

$\lim_{x\to\infty}\frac{1}{1+a^x}=\begin{cases}1\ (a<1)\\0\ (a>1)\end{cases}$ $\quad\lim_{x\to+\infty}(x-a\ln x)=+\infty$

INFINITESIMI EQUIVALENTI (*per* $x\to0$)

$\sin\alpha\sim\alpha$ $\quad\operatorname{tg}\alpha\sim\alpha$ $\quad 1-\cos\alpha\sim\alpha^2/2$

$\arcsin\alpha\sim\alpha$ $\quad\operatorname{arctg}\alpha\sim\alpha$ $\quad a^\alpha-1\sim\alpha\ln a$

$\ln(1+\alpha)\sim\alpha$ $\quad e^\alpha-1\sim\alpha$ $\quad(1+\alpha)^k-1\sim k\alpha$

DERIVATE NOTEVOLI

$f(x)$	$f'(x)$	$f(x)$	$f'(x)$
k	0	a^x	$a^x\ln a$
x	1	e^x	e^x
$ax+b$	a	$\ln x$	$\frac{1}{x}$
x^α	$\alpha x^{\alpha-1}$	$\log_a x$	$\frac{1}{x\ln a}$
$\frac{1}{x}$	$-\frac{1}{x^2}$	$\log_x a$	$-\frac{\ln a}{x\ln^2 x}$
$\sqrt{x}$	$\frac{1}{2\sqrt{x}}$	x^x	$x^x(1+\ln x)$
$x^{m/n}=\sqrt[n]{x^m}$	$\frac{m}{n}\sqrt[n]{x^{m-n}}$	$x(\ln x-1)$	$\ln x$
$\sin x$	$\cos x$	$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\cos x$	$-\sin x$	$\arccos x$	$-\frac{1}{\sqrt{1-x^2}}$
$\operatorname{tg}x$	$1/\cos^2 x=$ $=1+\operatorname{tg}^2 x$	$\operatorname{arctg}x$	$\frac{1}{1+x^2}$
$\operatorname{tg}x-x$	$\operatorname{tg}^2 x$	$\arccos\frac{-x}{\sqrt{1+x^2}}$	$\frac{1}{1+x^2}$
$\frac{1-\cos x}{1+\cos x}$	$\frac{\operatorname{tg}x/2}{\cos^2 x/2}$	$\frac{1-\operatorname{tg}x}{1+\operatorname{tg}x}$	$\frac{2}{1+\sin 2x}$

Regole di derivazione

- *Combinazione lineare*
$$D[a_1f_1(x)+\ldots+a_nf_n(x)]=a_1f_1'(x)+\ldots+a_nf_n'(x)$$
- *Prodotto*
$$D[f(x)\cdot g(x)]=f'(x)\cdot g(x)+f(x)\cdot g'(x)$$
- *Potenza*
$$D[f(x)]^\alpha=\alpha[f(x)]^{\alpha-1}f'(x)$$
- *Reciproca*
$$D\frac{1}{f(x)}=-\frac{f'(x)}{[f(x)]^2}$$
- *Quoziente*
$$D\frac{f(x)}{g(x)}=\frac{f'(x)\cdot g(x)-f(x)\cdot g'(x)}{[g(x)]^2}$$
- *Composta*
$$D[h\circ\ldots g\circ f(x)]=Dh(\ldots g(f(x)))=$$
$$=h'(\ldots g(f(x)))\cdot\ldots\cdot g'(f(x))\cdot f'(x)$$
- *Inversa*
$$Df^{-1}(y)=\frac{1}{f'(x)}\Leftrightarrow f'(x)=\frac{1}{Df^{-1}(y)}$$
- *Esponenziale*
$$D[f(x)]^{g(x)}=[f(x)]^{g(x)}\left(\frac{g(x)\cdot f'(x)}{f(x)}+g'(x)\cdot\ln f(x)\right)$$

DERIVATE N-SIME FONDAMENTALI

$f(x)$	$f^{(n)}(x)$
x^α	$\alpha(\alpha-1)\ldots(\alpha-n+1)x^{\alpha-n}$
x^n	$n!$
$a_0x^n+a_1x^{n-1}+\ldots+a_n$	$a_0\cdot n!$
a^x	$a^x(\ln a)^n$
$\sin x$	$\sin(x+n\pi/2)$
$\cos x$	$\cos(x+n\pi/2)$
$\sin\alpha x$	$\alpha^n\sin(\alpha x+n\pi/2)$
$\cos\alpha x$	$\alpha^n\cos(\alpha x+n\pi/2)$

TEOREMA DI DE L'HÔPITAL

Forma indet.	*trasformazione*	
$\frac{0}{0},\frac{\infty}{\infty}$	$\lim_{x\to x_0}\frac{f(x)}{g(x)}=\lim_{x\to x_0}\frac{f'(x)}{g'(x)}=\ldots=\lim_{x\to x_0}\frac{f^{(n)}(x)}{g^{(n)}(x)}$ ∃:stop	
$0\cdot\infty$	$\frac{0}{0}$	$\lim_{x\to x_0}[f(x)\cdot g(x)]=\lim_{x\to x_0}f(x)/(1/g(x))$
$+\infty-\infty$	$\frac{\infty}{\infty}$	$\lim_{x\to x_0}[f(x)-g(x)]=\lim_{x\to x_0}f(x)\left[1-\frac{g(x)}{f(x)}\right]$
$0^0,1^\infty,\infty^0,0^\infty$	$0\cdot\infty$	$\lim_{x\to x_0}[f(x)]^{g(x)}=\lim_{x\to x_0}e^{g(x)\ln f(x)}\ \ (f(x)>0)$
$\lg_0 0,\lg_0\infty,$ $\lg_\infty 0,\lg_\infty\infty$	$\frac{\infty}{\infty}$	$\lg_a b=\frac{\ln b}{\ln a}$
$\lg_1 1$	$\frac{0}{0}$	