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Accurate Prediction of the Three Dimensional Dispersion Characteristics of Loudspeaker Arrays Composed of Real or Theoretical Sound Sources

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Traditionally, the problem of predicting the behavior of loudspeaker arrays has been approached by making simplifying assumptions about the individual array elements. Assuming omnidirectional or piston behavior, or simplifying or disregarding phase response, can lead at best to good approximations of actual array behavior, and at worst to serious errors. A new graphics-based array simulation program has been developed which allows four traditional techniques as well as a new hybrid technique to be used in predicting the behavior of arbitrarily configured arrays. The correlation between the actual and predicted behavior of three test arrays is presented. Results show that the hybrid technique, based on measurements of an array element's full-sphere magnitude and phase response, is the most accurate predictor over the broad range of transducer and array types tested. A second technique, based on assumed phase response, is shown to be accurate in cases where the element's acoustic center is fixed. Three other traditional prediction techniques are shown to have limitations which can lead to significant errors.

0. INTRODUCTION

New emphasis has been placed on the role that loudspeaker dispersion plays in the success or failure of sound systems designed for large audiences. For example, the dispersion of a sound source can be responsible for poor speech intelligibility if it causes echoes or too many late-arriving reflections. Similarly, erratic dispersion can be the cause of uneven frequency response over an audience area. Clearly the goal of even sound distribution over a large audience area depends in part on the dispersion characteristics of the sound sources.

If a source's dispersion characteristics are known, a variety of microcomputer-based software programs can be used to predict the sound arriving at a listener location. The usefulness of these programs is limited, however, by the number of sources whose dispersion has been characterized. While the dispersion of many single loudspeaker types is known, the dispersion of loudspeaker arrays composed of two or more of these elements is usually not.

Though the need to know the three-dimensional dispersion characteristics of multiple-element arrays is compelling, acquiring the information involves difficult, unproven, or inaccurate techniques. These techniques are generally based on measurements or predictions. Careful measurements guarantee accurate information, but are time consuming and require specialized equipment and personnel. Even when the measurements have been made, they are rarely shared with sound system designers. These difficulties have made techniques for predicting array behavior more attractive to the designer. Prediction techniques, if they can be proven accurate, offer clear advantages over measurement techniques since they allow array optimization to take place inexpensively and with minimal risk.

In this study, the various techniques currently used to predict array behavior are reviewed. A new hybrid prediction technique is presented which combines measurement of individual array elements with a straightforward complex pressure summation scheme. An experiment has been designed to test the accuracy of the various techniques by comparing predictions to the actual behavior of three arrays. By averaging the errors between actual and predicted dispersion, the overall accuracy of the various predictors can be computed. Examples illustrating the major sources of error are also presented.

1. ARRAY PREDICTION TECHNIQUES

1.1 Basic Mathematics

Array prediction techniques require the summation of pressure contributions from each of the individual elements. This summing function is modified in important ways by the assumptions inherent in the various techniques. The basic mathematics for complex pressure summation from multiple sources is presented first, followed by the simplified summation formulae of the various prediction techniques.

The geometry governing an arbitrary array of elements is shown in Fig.-1a. Henceforth, the sphere centered at the array center will be called the *polar sphere*. The term *polar circle* will be used to describe the intersection between the polar sphere and any plane which contains the array center and the sphere's pole, as shown in Fig.-1b. The set of polar circles includes the circular paths traveled to generate traditional polar plots. The equation for computing the overall pressure at some point on the polar sphere is:

$$P(\theta,\varphi,f,R) = \sum_{n=1,N} p_n(\theta,\varphi,f,r_{n,\theta,\varphi}) \qquad \text{[Eqn.-1]}$$

where,

$P(\theta,\varphi,f,R)$	is the total complex pressure at frequency f and at distance R from the array center, and at the polar angle defined by θ and φ.
R	is the distance from the point on the polar circle to the array center.
N	is the number of sources.
$p_n(\theta,\varphi,f,R)$	is the complex pressure contribution from the n'th source, and can be expanded to:

$$p_n(\theta,\varphi,f,r_{n,\theta,\varphi}) = (a_{n,\theta,\varphi} / r_{n,\theta,\varphi}) * \exp[-j(kr_{n,\theta,\varphi} + \phi_{n,\theta,\varphi})] \qquad \text{[Eqn.-2]}$$

where,

$a_{n,\theta,\varphi}$	is the real pressure amplitude of the n'th source at polar angle θ and φ and some reference distance (usually one meter).
$\phi_{n,\theta,\varphi}$	is the phase in radians of the n'th source at θ and φ and the same reference distance.
k	is the "wave number" and is equal to $(2\pi f)/c$ where c is the wave speed.
$r_{n,\theta,\varphi}$	is the distance from the point on the polar sphere at angle θ and φ and radius R to the reference mark on the n'th source.

1.2 Simple Source Array Prediction Technique

The simple source technique assumes that each source is omnidirectional and has phase response the result of propagation delay only. The assumptions of this model are realistic when the source is much smaller than a wavelength of sound. The simple source model has been extended by several investigators to regions of higher frequency with various degrees of success [1-5]. In the case of loudspeakers, the technique has been shown to be useful in predicting the major features of dispersion, but has not been used to compare predictions with actual measurements to verify its accuracy. Because loudspeakers typically operate in regions where wavelengths are on the same order as or smaller than the sources, it can be expected that actual array behavior will deviate from predictions made using this technique.

The equation used to calculate dispersion using the simple source model is:

$$P(\theta,\varphi,f,R) = \sum_{n=1,N} p_n(\theta,\varphi,f,r_{n,\theta,\varphi}) \qquad \text{[Eqn.-3]}$$

where,

$$p_n(\theta,\varphi,f,r_{n,\theta,\varphi}) = (K_n / r_{n,\theta,\varphi}) * \exp[-jkr_{n,\theta,\varphi}] \qquad \text{[Eqn.-4]}$$

where,

K_n is the real and constant amplitude of the n'th source.

1.3 Power Sum Technique

The power sum technique assumes that phase interaction between elements can be disregarded. In this model, only the magnitudes of the energy contributions from the various elements are added. While phase interaction is responsible for many forms of array behavior, (a simple dipole is an example) there are situations where the interaction between elements is assumed to be negligible. This is an assumption usually made in designing horn clusters, for example.

The equation used to calculate dispersion using the power sum model is:

$$P^2(\theta,\varphi,f,R) = \sum_{n=1,N} |p_n(\theta,\varphi,f,r_{n,\theta,\varphi})|^2 \qquad \text{[Eqn.-5]}$$

where,

$$|p_n(\theta,\varphi,f,r_{n,\theta,\varphi})|^2 = [a_{n,\theta,\varphi} / r_{n,\theta,\varphi}]^2 \qquad \text{[Eqn.-6]}$$

$P^2(\theta,\varphi,f,R)$ is the total sound energy at frequency f and at radius R from the array center, and at polar angle defined by θ and φ.

1.4 Piston Source Technique

Another technique assumes that an array element can be modeled as a piston in an infinite baffle [6]. This technique is particularly attractive since the pressure can be computed from a closed form equation. Many authors have used the piston model to predict array behavior, but to this author's knowledge none has correlated prediction with actual measured behavior.

The equation used to calculate dispersion using the piston source model is:

$$P(\theta,\varphi,f,R) = \sum_{n=1,N} p_n(\theta,f,r_{n,\theta,\varphi}) \qquad \text{[Eqn.-7]}$$

where,

$$p_n = (j\rho_0 ckl_n U_n)/r_{n,\theta,\varphi}) * [J_1(kl_n \sin\theta_n)/(kl_n \sin\theta_n)] * \exp[-jkl_n] \qquad \text{[Eqn.-8]}$$

where,

ρ_0	is the density of air.
l_n	is the radius of the n'th piston source.
U_n	is the velocity amplitude of the n'th source.
J_1	is the Bessel function of the first kind.
θ_n	is the angle between the line from the point on the polar circle and the main axis of the n'th source, and is bounded by -90° < θ < +90°.

The assumption that the piston is located in an infinite baffle restricts predictions to the region -90°< θ<+90°; rear radiation is not predicted. The piston model is useful for cone-type loudspeakers operating in frequency regions where each part of the cone is moving with the same velocity and phase. The piston model is not useful for horn sources or other sources which have behavior fundamentally different from pistons.

1.5 Phasor Sum Technique

Another technique computes the phase and magnitude sum from an array of sources but assumes that the phase response is due solely to propagation delay. In fact, this "phasor" or vector sum is identical to the simple source model with the exception that it does not assume omnidirectionality. The originators of this technique [7] state that the phasor sum should be restricted to arrays of like devices. The technique is not recommended for predicting the dispersion characteristics of any cluster made up of different devices. In these cases, the power sum model is recommended in [7].

The equation used to calculate dispersion using the phasor sum technique is:

$$P(\theta,\varphi,f,R) = \sum_{n=1,N} p_n(\theta,\varphi,f,r_{n,\theta,\varphi}) \qquad \text{[Eqn.-9]}$$

where,

$$p_n(\theta,\varphi,f,r_{n,\theta,\varphi}) = (a_{n,\theta,\varphi}/ r_{n,\theta,\varphi}) * \exp[-jkr_{n,\theta,\varphi}] \qquad \text{[Eqn.-10]}$$

1.6 Finite Element Models

Other investigators have worked to predict the dispersion characteristics of the individual elements theoretically [8-10]. These predictions are usually based on a finite element model of the individual elements. While some of these models appear promising, none has emerged which is suitable for predicting the phase and magnitude behavior of the broad range of transducer types used in sound reinforcement.

2. HYBRID TECHNIQUE FOR PREDICTING ARRAY BEHAVIOR

The fundamental limitations of existing techniques led us to a new approach. In this approach, the simplifying assumptions made by the simple source, piston, power sum, and phasor sum models are avoided through the use of comprehensive measurements of array elements. Once array elements have been measured, prediction of array behavior is carried out on a new software program. Calculation of array behavior is made using a traditional complex pressure summation scheme described in Section-1.1 above. This hybrid approach is suitable for use in predicting the behavior of most types of modern arrays, from horn clusters to arrays of wide-bandwidth drivers.

2.1 Measurement of Array Elements

Measurement of individual array elements is based on the need to capture both phase and magnitude information over the full polar sphere. The measurement setup used to measure array elements is shown in Fig.-2. In this setup a computer-based dual-channel FFT system generates a time-gated noise burst whose exact magnitude and phase is measured along with the signal arriving from the microphone. These two stored signals are transformed to the frequency domain and converted to magnitude and phase response. The exact phase relationship between the source and the microphone is preserved by ensuring that a single reference point marked on the source is always directly over the center of turntable rotation.

For each element, and at each location on the polar circle, a 4,096-point frequency response curve was computed over a bandwidth of 0-10 kHz. Sixty four separate files were generated for a 360° rotation of the turntable, corresponding to a polar resolution of 5.625°. A signal to noise ratio of at least 40dB, and power compression of less than 0.5 dB were ensured over the passband of the speaker for all measurements.

2.2 Data Reduction

The measurements described above yield 64 x 4096 = 262,144 complex numbers for a single polar plane. For full polar sphere measurements, 18 x 262,144 ≈ 4.7 million complex numbers are needed. Thus the data storage, dynamic memory, and calculation speed of the best microcomputers forced the reduction of the measurement database to a more manageable one.

Data reduction is possible in the spatial and spectral domains. In general, a source whose response (either spatially or spectrally) is relatively smooth and gradual can tolerate more data reduction than a source which has an erratic or rapidly changing response. For the three transducers used in this study, it was found that spatial resolution could be reduced to 10° crossings of longitude and latitude, and spectral resolution could be reduced to one-third octave bands. (As will be shown below, the reduction of *measured* data to 10° and one-third octave resolution does not preclude the ability to *predict* high-resolution spectral or spatial behavior.)

2.3 Program for Predicting Array Behavior

The software program written for the purpose of predicting array behavior is called *ArrayCAD* [11] and runs on the Apple Macintosh [12] family of computers. The program is designed to allow the rapid construction and modification of array models through the use of an advanced user interface.

The three-dimensional shape of each sound source is represented by a wire-frame model. Source acoustics are represented by full-sphere magnitude and phase response and on-axis sensitivity. Individual array elements can be placed and aimed into an array using Cartesian or spherical coordinate systems. Each array element is defined by an "electrical" driving signal consisting of amplitude, delay, and frequency response thus allowing any of the well known array dispersion control techniques to be used such as amplitude, phase, frequency and Bessel shading schemes [13].

Prediction of array behavior is based upon the principle of superposition. The user can choose between simple source, power sum, piston source, and phasor sum techniques, or the new hybrid technique.

The user can predict either sine-wave or constant-bandwidth response; for example, predictions can be made at 2,000 Hz, or in the one-third octave band whose center frequency is 2,000 Hz. In addition, the user can select the number of frequencies representing a given band; for example, the one-third octave band centered at 2 kHz can be represented by 10 equally spaced frequencies between 1,748 and 2,245 Hz.

Results are presented as either full-sphere polar response, individual polar plots, or frequency response at a specified point on the polar sphere, as illustrated in Figs.-3-5. Resolution of predicted polar response is selectable from 0.25-10.0°.

The program currently has no ability to predict the diffraction effects caused by nearby array elements or to any objects used to build the array. While it is recognized that in some real situations these effects can be significant, modeling them is extremely complicated and has been reserved for future investigation.

3. TEST ARRAYS

In order to test the accuracy of the various prediction techniques, it was necessary to construct and measure real arrays. For this purpose three arrays, each using different transducers, were chosen. Measurements were made on the setup shown in Fig.-2. For the two arrays composed of cone speakers, measurements were made with 5.625° resolution. For the array composed of horns, where narrow-angle interference effects are known to occur [14-16], a much higher resolution was used, corresponding to about 0.5°.

3.1 Twiddler [17] Array

The driver used in this array is a wide bandwidth 2.25" (11-cm) diameter speaker. The driver is housed in a 45-in³ (730-cm³) sealed enclosure. Two of these units were used to create a simple array where the centers of the drivers were separated by 4.7" (12-cm), as shown in Fig.-6.

3.2 4.5" Driver Array

The driver used in the second array is a very wide bandwidth 4.5" (11-cm) diameter loudspeaker. The array consisted of four drivers located in a line and angled as shown in Fig.-7. While the baffle used in this experiment is the same as that employed in the Bose model 402 loudspeaker, the electronic shading circuit used in the 402 to control dispersion at high frequencies was removed. Therefore the polar data presented for this 4.5" driver array does not resemble that of the 402 speaker.

3.3 Horn Array

The third test array consisted of two Electro-Voice HP-6040A [12] constant directivity horns driven by DH2 compression drivers. The horns were aimed such that their patterns overlapped at their -6dB points at 2 kHz. This corresponded to splay angles of ±25°. The horns were placed so as to ensure that their drivers lay on the surface of a sphere, and were as close together as possible, as shown in Fig.-8.

3.4 Array Models

For each of the three test arrays, an array model was built in the ArrayCAD program. Array elements were placed in models exactly as they existed when measured. Once the array models were built, the program was configured to predict dispersion using the various prediction techniques.

For this study, predictions were performed over the ten one-third octave band frequencies ranging from 500-4000 Hz and in the horizontal polar plane. A given one-third octave band was divided into ten separate frequencies and array prediction was based on a complex sum of all ten frequencies. In the case of the Twiddler and 4.5" arrays, dispersion was calculated using a polar resolution of 10°. The program was configured to predict polar response in 1° resolution in the case of the horn array in an attempt to predict any narrow-angle interference effects.

4. RESULTS

4.1 Overall Accuracy of Prediction Techniques

For each array and for each prediction technique, a series of ten polar plots corresponding to the ten one-third octave bands were generated. For each of the three arrays, this results in a total of 3 x 5 x 10 = 150 polar plots. A method of reducing the polar data was used whereby only polar data within 12-dB of the axial sound pressure level was considered. In this method, if both the actual and predicted polar data were more than 12-dB down compared to the on-axis level, the data were discarded. This choice was made in order to compare the accuracies of the predictors in the range most critical to the sound system designer; an error between actual and predicted data when both pressure levels were more than 12 dB down is much less important than an error found within the 12dB range.

The errors between the predicted and actual polar responses at each prediction point around the ten polar plots were averaged to yield a single value representative of the accuracy of the prediction technique. Thus the higher the average error, the less accurate the predictor. The overall results using this averaging method are shown in Figs.-9-11.

From the overall results, it can be seen that the new hybrid technique has low overall error for each of the three different arrays. The phasor sum technique is equally accurate except in the case of the horn array, where it is slightly less accurate than the hybrid technique. The piston source technique is accurate in the spatial region and transducer type to which it is restricted. The power sum and simple source techniques are significantly less accurate.

4.2 Illustrative Examples

While the averaging method described in Section-4.1 indicates the overall accuracy of the various prediction techniques, it does not show the major sources of error. Closer examination of the polar data can be used to expose these sources.

4.2.1 Twiddler and 4.5" Driver Arrays

In the case of the Twiddler array, the size of the drivers is small enough that relatively wide dispersion results even up to 4000 Hz. This allows significant interference to take place between the two drivers. The spacing of the two speakers reaches the equivalent of one half wavelength at about 2500 Hz, causing dipole-like behavior. The power sum technique, with its neglect of phase interaction, will completely miss the strong interaction at these frequencies. The simple source technique, while accurately predicting the major interference behavior of the spaced sources, results in serious errors due to its assumption that the sources are omnidirectional.

Both the piston and phasor techniques can be expected to predict the Twiddler array behavior accurately since their basic assumptions are mostly valid in the frequency range used here. The modest error observed in the piston technique relative to the phasor sum technique may be due to the fact that at the higher frequencies the driver does not behave like a simple piston, and secondly, to the incorrect assumption that the driver is mounted in an infinite baffle. Side by side polar plots illustrating these effects in the 2500 Hz band are shown in Fig.-12.

A similar situation exists for the 4.5" driver array. Again, the assumption of no phase interaction in the power sum technique, and the assumption of omnidirectionality in the simple source technique result in serious errors. The accuracy of the piston, phasor, and hybrid techniques are virtually identical, although errors are several dB higher than the Twiddler array.

4.2.2 Horn Array

In the case of the horn array, both the hybrid and phasor sum techniques were able to predict the very narrow-angle interference effects measured on the actual horn array. The power sum technique is incapable of predicting these phase-based interference effects. The ability (or inability) to predict these narrow-angle effects accounts for most of the difference in accuracy between the three techniques.

The hybrid technique is slightly more accurate than the phasor technique, due primarily to the ability to predict more exactly the angle of the interference nulls. Because the new technique relies on measured phase information while the phasor sum technique relies on assumed phase response, the location of these nulls can be expected to be predicted more accurately by the new technique. Fig.-13 shows measurements and predictions illustrating these trends.

5. DISCUSSION

5.1 Similarity in Accuracies of Hybrid and Phasor Techniques

Results comparing the accuracy of the different array behavior prediction techniques indicate that the new hybrid technique is only slightly more accurate than the phasor technique. The only difference between the two techniques is the use of measured versus assumed phase response. The phasor technique assumes that phase response is a result of propagation delay only. If the actual phase response of an array element can be shown to be due to propagation delay only, then the primary assumption of the phasor model will have been proven. The conditions for propagation-time only phase response are shown in Fig.-14, where it is assumed that the acoustic center is placed somewhere near to, but not exactly coincident with the turntable axis of rotation, and, therefore,

$$\phi_T = \phi_R + \phi_\theta = K - kr\cos\varphi$$

where,

ϕ_T is the total measured phase.

ϕ_R is the phase due to delay from center of rotation to mic.

ϕ_φ	is the phase due to acoustic center not being exactly aligned to center of rotation.
φ	is the angle of rotation of the turntable.
K	is a constant
k	is the "wave number" and is equal to $2\pi f / c$.

Under these conditions, the acoustic center simply rotates about the turntable axis of rotation, tracing out a small circle, and thus giving rise to a sinusoidally varying delay time or phase from the acoustic center to the measurement microphone. Thus, one complete revolution of the turntable corresponds to one sinusoidal period in the phase versus angle response.

All three of the array elements used in this study -- the Twiddler, the 4.5" driver and the constant directivity horn (in the horizontal plane only), revealed phase responses which did not significantly deviate from sinusoidal, although this result was not expected.

A stable acoustic center, therefore, is thought to be primarily responsible for the similarities in accuracy of the the hybrid and phasor techniques. If array elements were used whose acoustic centers were not fixed, it could be expected that the hybrid prediction technique would continue to be accurate because it includes the non-sinusoidal phase response, while the phasor technique would become less accurate. It can be expected, for example, that the phase response of a horn speaker would deviate from sinusoidal on non-orthogonal polar planes, where the device becomes non-symmetric with angle around the polar circle.

5.2 Extension of the Phasor Technique

The usefulness of the phasor technique has been significantly extended in this study. The authors who introduced the technique stated that it should be restricted to arrays of like devices and to single frequencies [7]. In this study, the phasor technique has been shown to be useful for bandwidths of at least one-third octave, and for a variety of different transducers. These results indicate that the phasor model is useful for arrays composed of elements whose phase response is known to be a result of propagation delay only, or in other words, for sources whose acoustic centers are known to be fixed.

5.3 Effectiveness of Data Reduction

The database used to characterize array elements from measured responses consisted of a single complex number for each 10° crossing of longitude and latitude lines on the measurement sphere. This corresponds to a database size of only 36 x 18 = 648 complex numbers for each one-third octave band. What is remarkable is that the narrow-angle interference effects known to exist in the overlap region between constant directivity horns can be predicted with excellent accuracy even with these low resolution array element databases. The low resolution database used to characterize array elements was sufficient to predict narrow-angle interference effects simply by high-resolution sampling in the array prediction program. This indicates that high-resolution measurements of array elements are not needed to predict complex yet highly significant array behavior patterns.

5.4 Effectiveness of Other Techniques Tested

The piston model, while shown to be useful for arrays using cone speakers, is not applicable to horn arrays or any other array element which differs fundamentally from a piston, and cannot be used to predict rear radiation in its simple form. It is possible that the accuracy of the piston model could be improved if a more exact solution for an unflanged piston [18] was exploited; unfortunately, this solution is not in closed form. This study indicates that the piston model is a good choice when neither magnitude nor phase data is available for the source in question.

The simple source model is of less value as a predictive technique, but nevertheless is capable of exposing the major front-radiation interference effects of spaced sources. The simple source model is probably of most value for predicting the behavior of low-frequency arrays. When the individual elements become directional, or have non-constant phase response as a function of angle the model will produce serious errors. In this study, most of the error associated with the simple source method is in the rear radiation region. Lastly, the simple source method is not a good method for predicting horn array behavior.

The power sum model is only useful in arrays where the individual elements have minimal interaction. In the broad range of frequencies tested here, both the Twiddler and 4.5" drivers interact strongly, and therefore the power sum model is seriously flawed. In the case of horn arrays, the success of the power sum model is dependent on the aiming of the individual horns. If the horns are aimed so that their -6dB points are aligned, significant interference results, and the power sum model will be unable to account for the interference effects. If horn elements are angled wider than their -6dB points, then the power sum model can be expected to be more accurate. However, this increased angling may not be practical in cluster design, since it is likely to create a hole or dark spot in the horn array's polar response.

Finally, it must be stressed that the power sum technique has the undesirable effect of masking the interference effects which occur in the proper aiming of constant directivity horns. These interference effects, while narrow in angle, are relatively deep and occur directly on the axis between the two horns where it is likely to be in line with listeners. Fig.-15 shows the frequency response 10° off axis of the horn array, compared to a power sum of the two horns in the same array. The phase-based interference effects causing the uneven frequency response shown in Fig.-15 will have an effect on the evenness of sound distribution in a room, especially when the array is designed in such a way as to strictly minimize the reverberant field, as is traditionally the case with horn arrays. In these cases, undesirable array interference effects will be projected directly onto the audience.

6. CONCLUSION

The relative accuracies of five different methods used to predict the dispersion behavior of loudspeaker arrays has been tested against the actual behavior of three arrays. Results indicate that a hybrid technique proposed here, based on the measurement of both phase and magnitude over the entire polar sphere of an array element, is most promising as being applicable to all transducer and array types. The phasor sum technique, based on measured magnitude and assumed phase response, was also shown to be accurate. This technique, however, can be expected to produce errors whenever array elements do not meet the requirement of phase response due to propagation time only.

Other traditional prediction techniques were shown to have limitations and restrictions, but nevertheless are useful for predicting array behavior in some circumstances. The piston model is limited to array elements whose behavior resembles a piston, such as small cone drivers, but cannot be used to predict rear radiation from arrays, or for predicting the behavior of horn arrays. The simple source model is limited to low frequencies or, at higher frequencies, for predicting general front-radiation interference effects. And the power sum model is only useful in the case where array elements have negligible interaction.

Significantly, it was shown that only low-resolution, full-sphere polar response was required to predict narrow-angle interference effects. These predictions were made by implementing high-resolution sampling about the array model polar circle. The significance of this finding is that high resolution measurements of actual array elements were not required for accurate prediction of detailed array behavior.

These results indicate that significantly increased accuracy in the prediction of array behavior requires the inclusion of phase information. If the new hybrid prediction technique described in this study can be generalized, manufacturers will need to publish both magnitude and phase information for their arrayable speakers, but they will only need to do this at moderate measurement resolution. At the very least, this study indicates the need for manufacturers to publish full-sphere polar response for their most common arrays.

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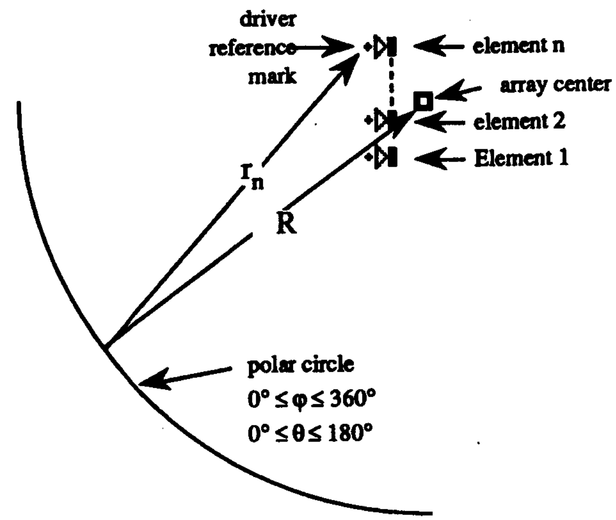

Fig.-1a Basic geometry for adding the individual pressure contributions from each element in an arbitrary array.

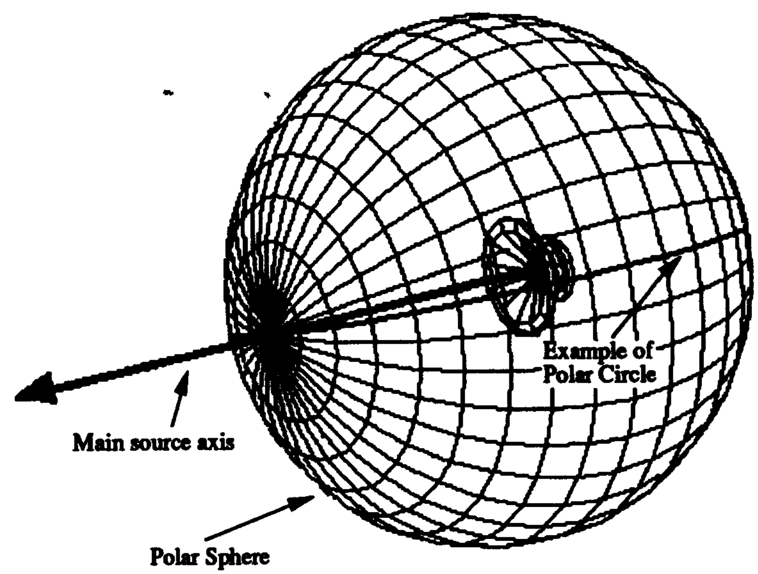

Fig.-1b Schematic of polar sphere, polar circle, and main source axis. Measurements/predictions take place at intersection points of longitude and latitude lines.

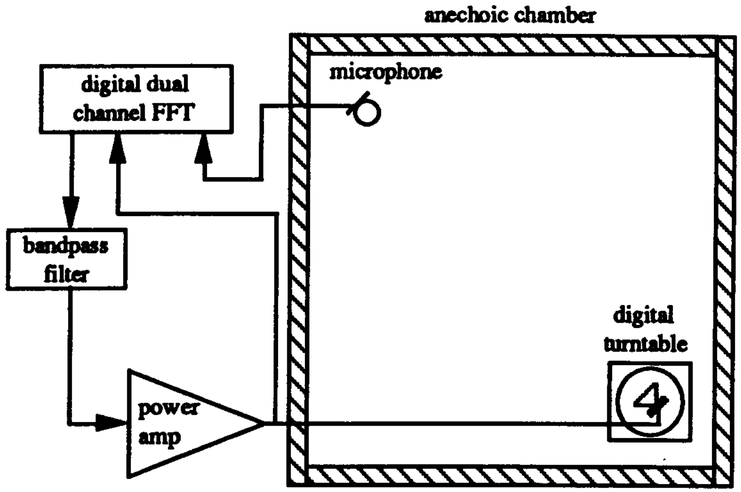

Fig.-2 Setup for measuring the full-sphere magnitude and phase polar response of an array element.

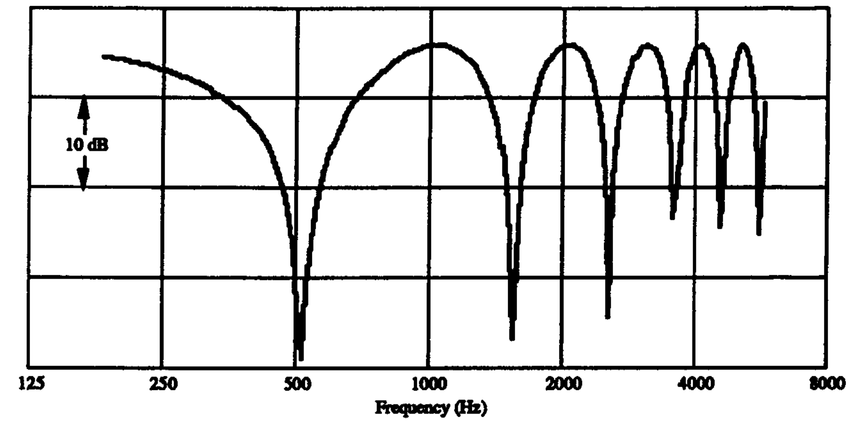

Fig.-4 Frequency response output from ArrayCAD program for the case of a simple dipole.

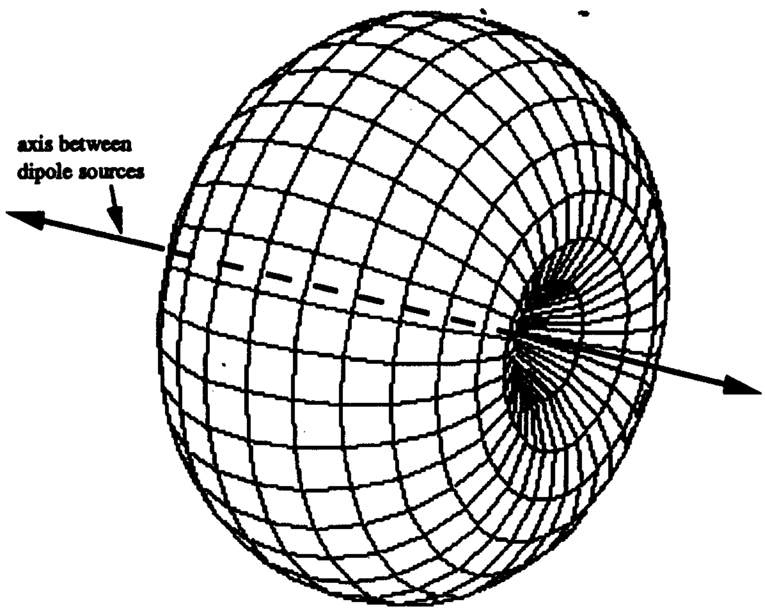

Fig.-3 Oblique view of full-sphere polar balloon resulting from a simple dipole. (Output is directly from the ArrayCAD program.)

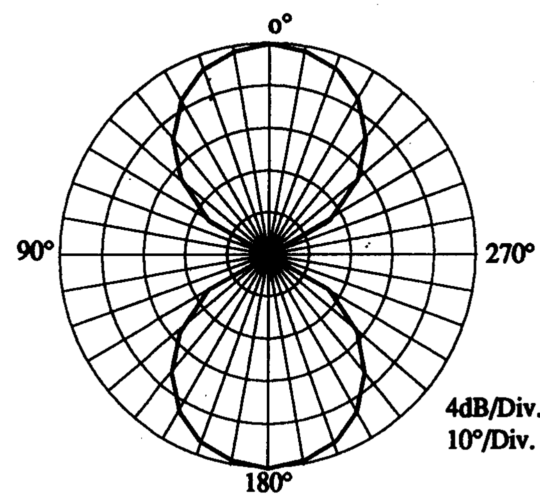

Fig.-5 Single plane polar plot output from ArrayCAD for the case of a simple dipole.

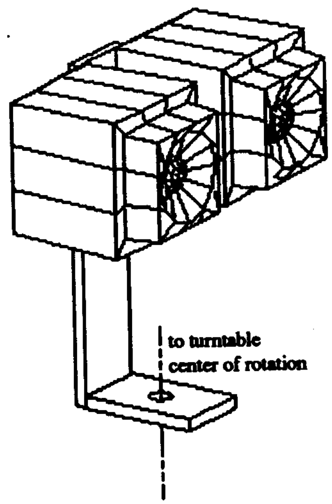

Fig.-6 Oblique view of Twiddler array from ArrayCAD program. Drivers centers are separated by 4.7" (12 cm).

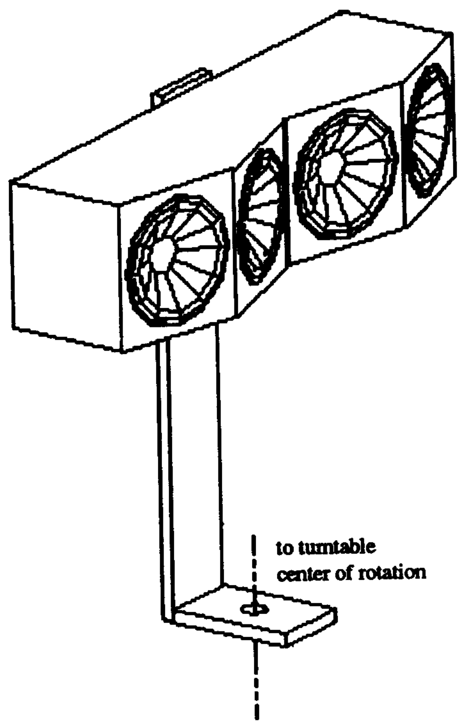

Fig.-7 Oblique view of 4.5" array from ArrayCAD program. Drivers centers are separated by 4.8" (12.2 cm) and are aimed at ±15° from center.

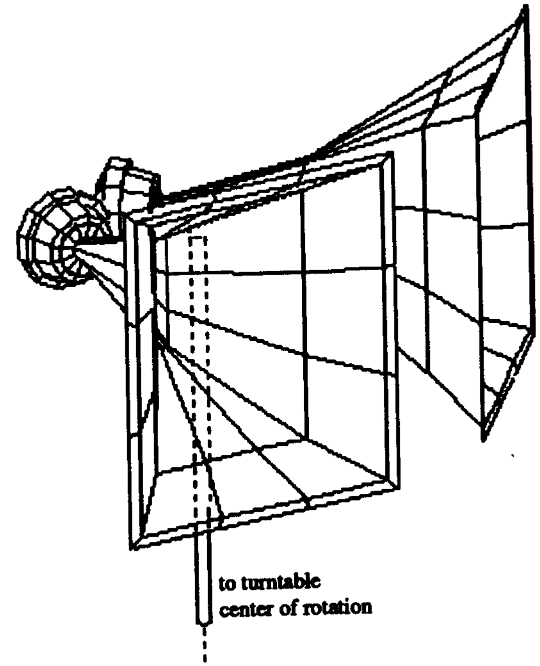

Fig.-8 Oblique view of horn array from ArrayCAD program. Horns are aimed at ±25° and are aligned so that the backs of the compression drivers fall on a sphere of radius ≈ 13.3" (35 cm).

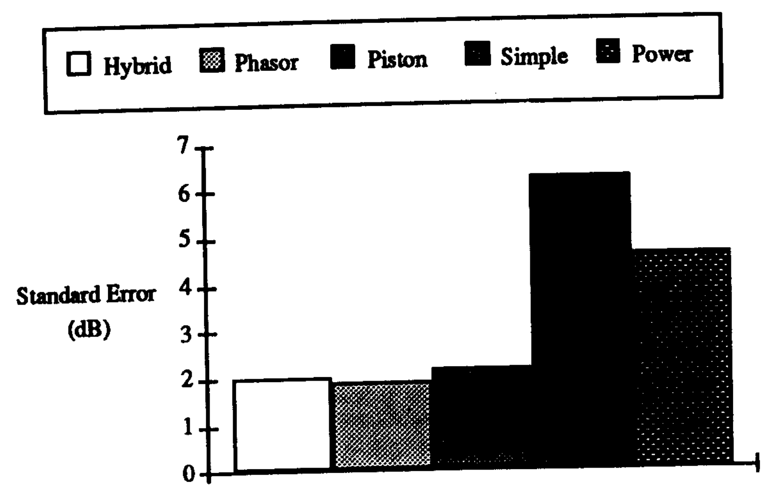

Fig.-9 Overall standard errors for five different array prediction techniques -- Twiddler array.

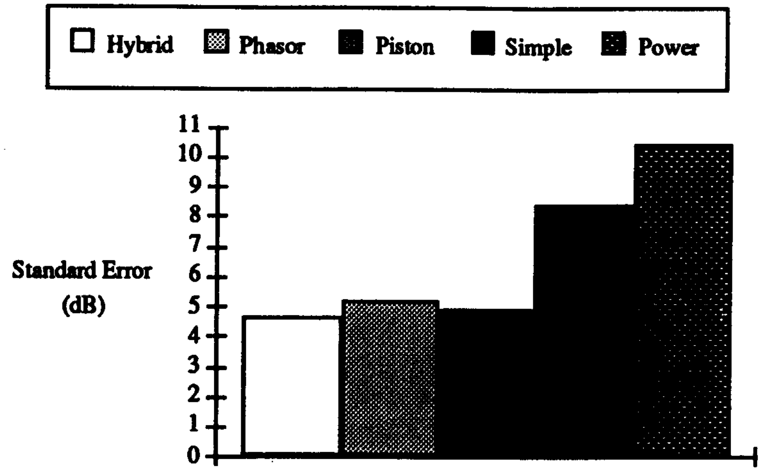

Fig.-10 Overall standard errors for five different array prediction techniques -- 4.5" driver array.

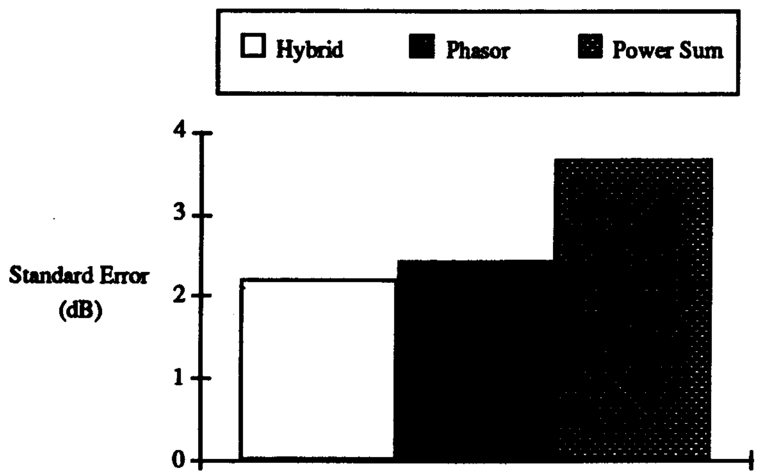

Fig.-11 Overall standard errors for three different array prediction techniques -- horn array.

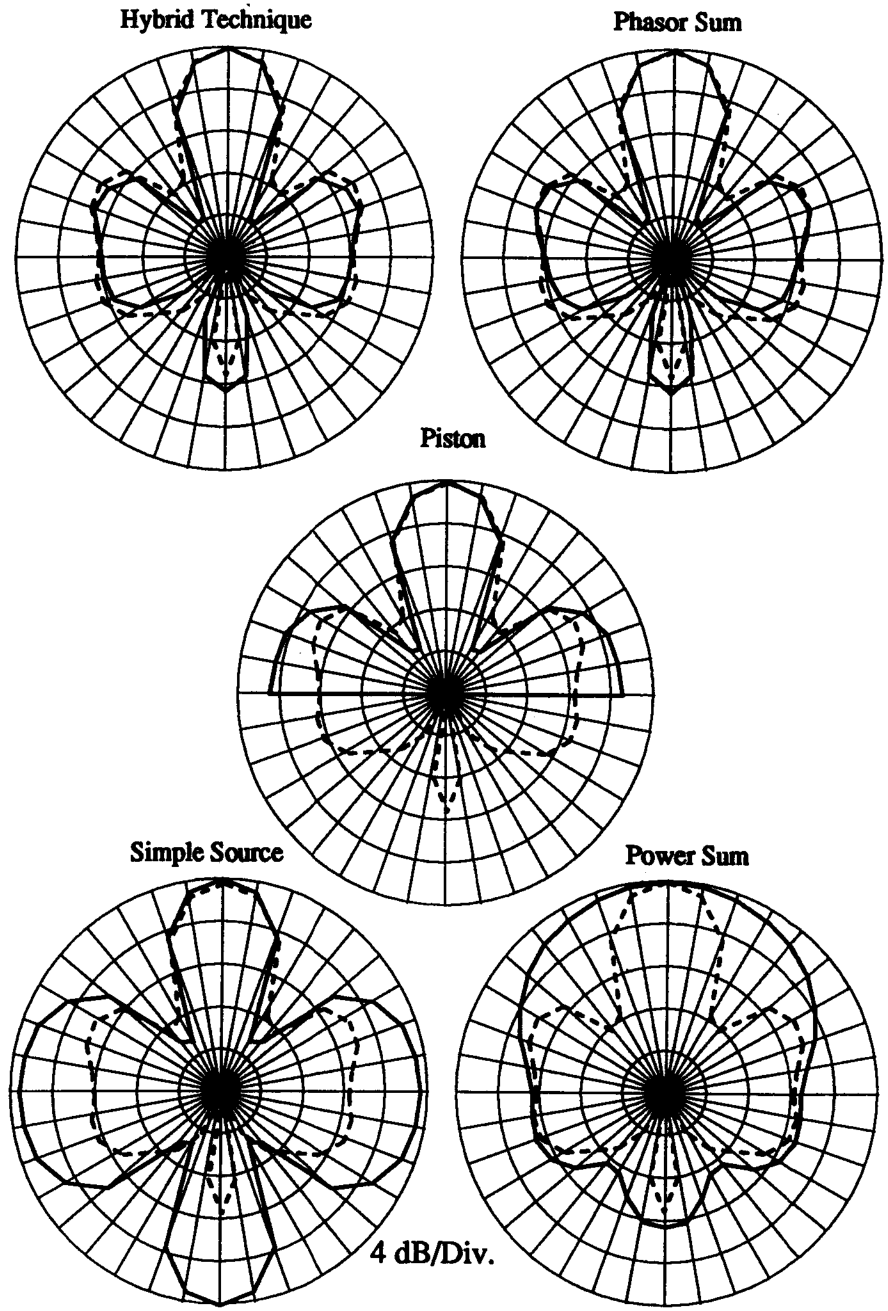

Fig.-12 Predicted (solid lines) vs. measured (dashed lines) Twiddler array behavior for five different array prediction techniques in 2500 Hz band.

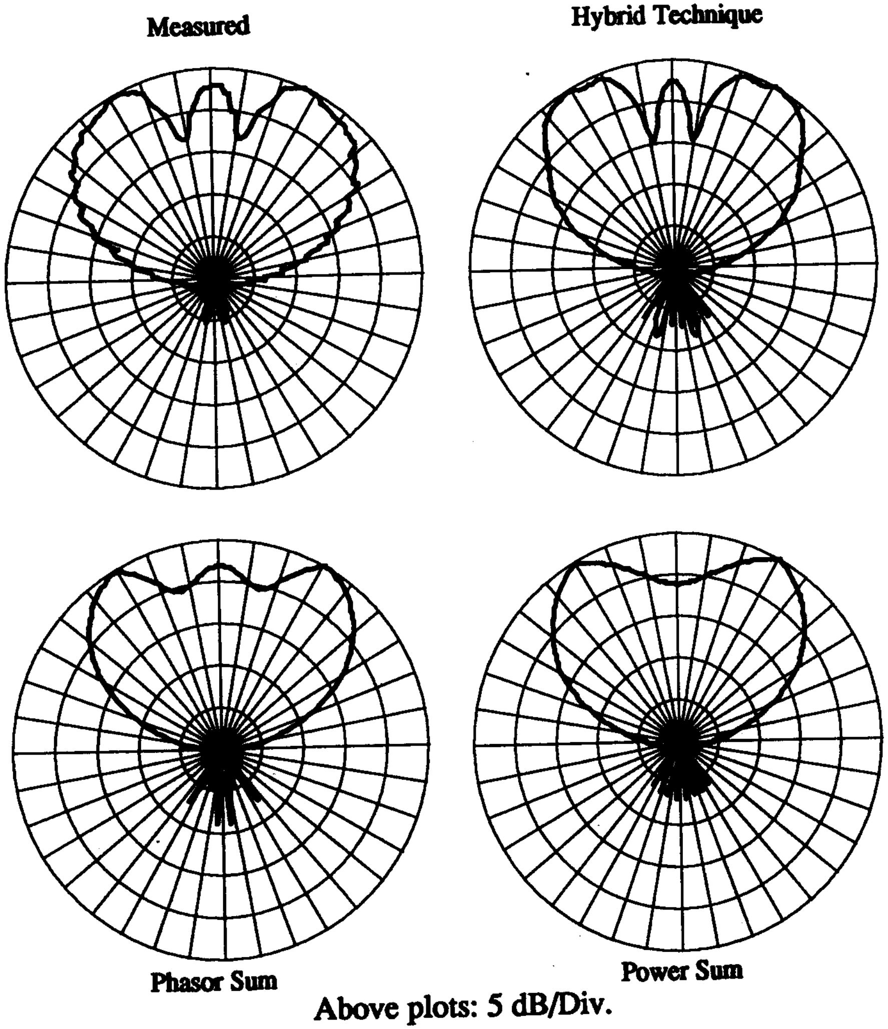

Fig.-13 Measured and predicted horn array behavior in 1000 Hz band for three different array prediction techniques. Notice that angle of dip near ±10° and off-axis region between 10 and 20° is better predicted by hybrid technique. Axial interference nulls deepen and migrate towards 0° as frequency increases (not shown).

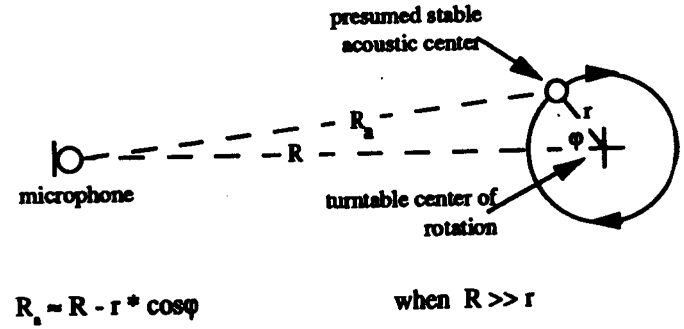

Fig.-14 Schematic diagram showing a fixed acoustic center travelling about the turntable axis of rotation. Phase versus φ response of such a system will be sinusoidal in shape.

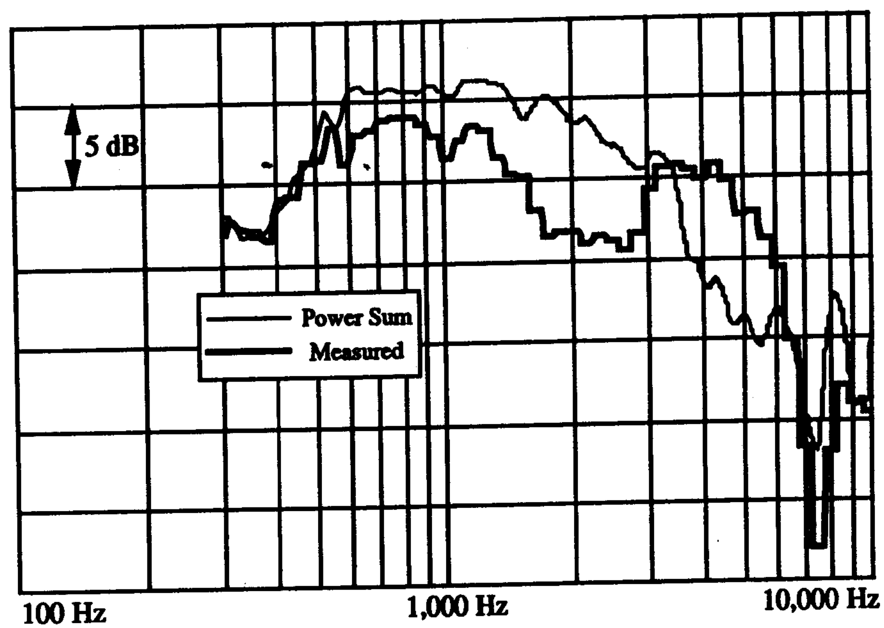

Fig.-15 Frequency response 10° off central axis of horn array. Heavy line is measured response. Light line is a power sum of two horns at the same angle. Deviation is due to interference effects not accounted for by power sum technique.