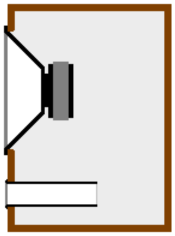
Acoustics/Bass-Reflex Enclosure Design

Introduction



Bass-reflex enclosures improve the low-frequency response of loudspeaker systems. Bass-reflex enclosures are also called "vented-box design" or "ported-cabinet design". A bass-reflex enclosure includes a vent or port between the cabinet and the ambient environment. This type of design, as one may observe by looking at contemporary loudspeaker products, is still widely used today. Although the construction of bass-reflex enclosures is fairly simple, their design is not simple, and requires proper tuning. This reference focuses on the technical details of bass-reflex design. General loudspeaker information can be found here.

Effects of the Port on the Enclosure Response

Before discussing the bass-reflex enclosure, it is important to be familiar with the simpler sealed enclosure system performance. As the name suggests, the sealed enclosure system attaches the loudspeaker to a sealed enclosure (except for a small air leak included to equalize the ambient pressure inside). Ideally, the enclosure would act as an acoustical compliance element, as the air inside the enclosure is compressed and rarified. Often, however, an acoustic material is added inside the box to reduce standing waves, dissipate heat, and other reasons. This adds a resistive element to the acoustical lumped-element model. A non-ideal model of the effect of the enclosure actually adds an acoustical mass element to complete a series lumped-element circuit given in Figure 1. For more on sealed enclosure design, see the Sealed Box Subwoofer Designpage.

Figure 1. Sealed enclosure acoustic circuit.

In the case of a bass-reflex enclosure, a port is added to the construction. Typically, the port is cylindrical and is flanged on the end pointing outside the enclosure. In a bass-reflex enclosure, the amount of acoustic material used is usually much less than in the sealed enclosure case, often none at all. This allows air to flow freely through the port. Instead, the larger losses come from the air leakage in the enclosure. With this setup, a lumped-element acoustical circuit has the following form.

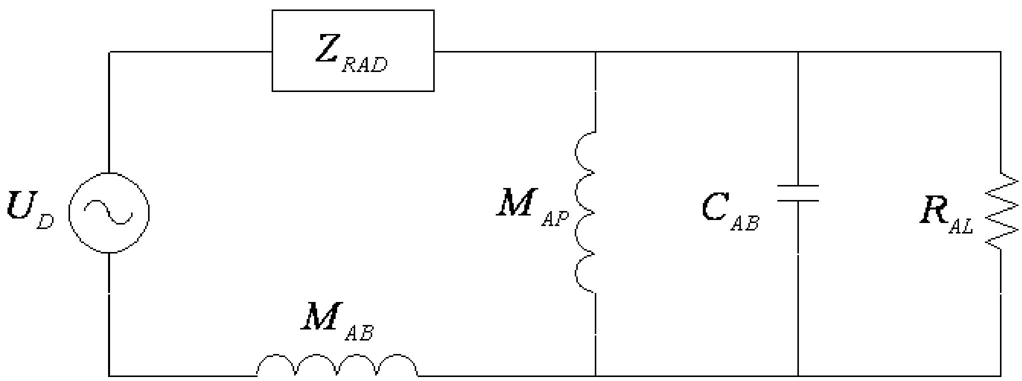


Figure 2. Bass-reflex enclosure acoustic circuit.

In this figure, Z_{RAD} represents the radiation impedance of the outside environment on the loudspeaker diaphragm. The loading on the rear of the diaphragm has changed when compared to the sealed enclosure case. If one visualizes the movement of air within the enclosure, some of the air is compressed and rarified by the compliance of the enclosure, some leaks out of the enclosure, and some flows out of the port. This explains the parallel combination of M_{AP}, C_{AB}, and R_{AL}. A truly realistic model would incorporate a radiation impedance of the port in series with M_{AP}, but for now it is ignored. Finally, M_{AB}, the acoustical mass of the enclosure, is included as discussed in the sealed enclosure case. The formulas which calculate the enclosure parameters are listed in Appendix B.

It is important to note the parallel combination of M_{AP} and C_{AB}. This forms a Helmholtz resonator (click here for more information). Physically, the port functions as the “neck” of the resonator and the enclosure functions as the “cavity.” In this case, the resonator is driven from the piston directly on the cavity instead of the typical Helmholtz case where it is driven at the “neck.” However, the same resonant behavior still occurs at the enclosure resonance frequency, f_B. At this frequency, the impedance seen by the loudspeaker diaphragm is large (see Figure 3 below). Thus, the load on the loudspeaker reduces the velocity flowing through its mechanical parameters, causing an anti-resonance condition where the displacement of the diaphragm is a minimum. Instead, the majority of the volume velocity is actually emitted by the port itself instead of the loudspeaker. When this impedance is reflected to the electrical circuit, it is proportional to 1 / *Z*, thus a minimum in the impedance seen by the voice coil is small. Figure 3 shows a plot of the impedance seen at the terminals of the loudspeaker. In this example, f_B was found to be about 40 Hz, which corresponds to the null in the voice-coil impedance.

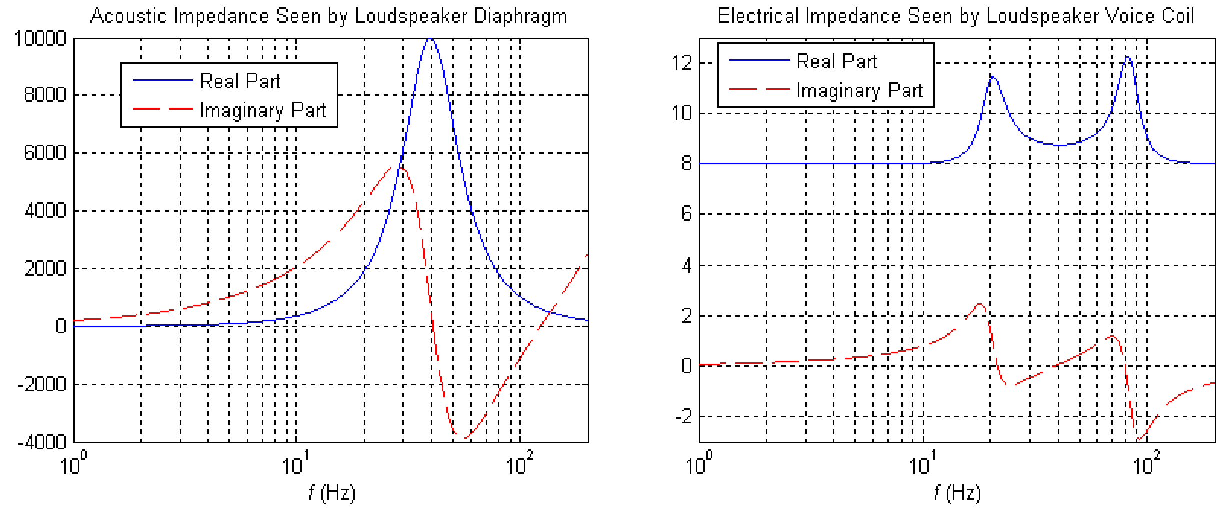


Figure 3. Impedances seen by the loudspeaker diaphragm and voice coil.

Quantitative Analysis of Port on Enclosure

The performance of the loudspeaker is first measured by its velocity response, which can be found directly from the equivalent circuit of the system. As the goal of most loudspeaker designs is to improve the bass response (leaving high-frequency production to a tweeter), low frequency approximations will be made as much as possible to simplify the analysis. First, the inductance of the voice coil, L_E, can be ignored as long as $\omega \ll R_E/L_E$. In a typical loudspeaker, L_E is of the order of 1 mH, while R_E is typically 8Ω, thus an upper frequency limit is approximately 1 kHz for this approximation, which is certainly high enough for the frequency range of interest.

Another approximation involves the radiation impedance, Z_{RAD}. It can be shown [1] that this value is given by the following equation (in acoustical ohms):

$$Z_{RAD} = \frac{\rho_0 c}{\pi a^2}\left[\left(1 - \frac{J_1(2ka)}{ka}\right) + j\frac{H_1(2ka)}{ka}\right]$$

Where $J_1(x)$ and $H_1(x)$ are types of Bessel functions. For small values of *ka*,

$$J_1(2ka) \approx ka \text{ and } H_1(2ka) \approx \frac{8(ka)^2}{3\pi} \Rightarrow Z_{RAD} \approx j\frac{8\rho_0\omega}{3\pi^2 a} = jM_{A1}$$

Hence, the low-frequency impedance on the loudspeaker is represented with an acoustic mass M_{A1} [1]. For a simple analysis, R_E, M_{MD}, C_{MS}, and R_{MS} (the transducer parameters, or *Thiele-Small* parameters) are converted to their acoustical equivalents. All conversions for all parameters are given in Appendix A. Then, the series masses, M_{AD}, M_{A1}, and M_{AB}, are lumped together to create M_{AC}. This new circuit is shown below.

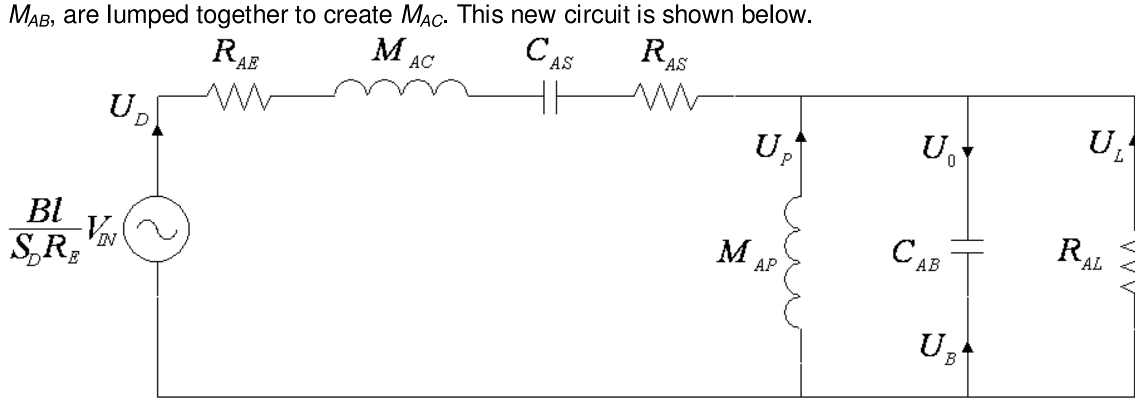


Figure 4. Low-Frequency Equivalent Acoustic Circuit

Unlike sealed enclosure analysis, there are multiple sources of volume velocity that radiate to the outside environment. Hence, the diaphragm volume velocity, U_D, is not analyzed but rather $U_0 = U_D + U_P + U_L$. This essentially draws a “bubble” around the enclosure and treats the system as a source with volume velocity U_0. This “lumped” approach will only be valid for low frequencies, but previous approximations have already limited the analysis to such frequencies anyway. It can be seen from the circuit that the volume velocity flowing *into* the enclosure, $U_B = -U_0$, compresses the air inside the enclosure. Thus, the circuit model of Figure 3 is valid and the relationship relating input voltage, V_{IN} to U_0 may be computed.

In order to make the equations easier to understand, several parameters are combined to form other parameter names. First, ω_B and ω_S, the enclosure and loudspeaker resonance frequencies, respectively, are:

$$\omega_B = \frac{1}{\sqrt{M_{AP}C_{AB}}} \omega_S = \frac{1}{\sqrt{M_{AC}C_{AS}}}$$

Based on the nature of the derivation, it is convenient to define the parameters ω_0 and h, the Helmholtz tuning ratio:

$$\omega_0 = \sqrt{\omega_B\omega_S}\, h = \frac{\omega_B}{\omega_S}$$

A parameter known as the *compliance ratio* or *volume ratio*, α, is given by:

$$\alpha = \frac{C_{AS}}{C_{AB}} = \frac{V_{AS}}{V_{AB}}$$

Other parameters are combined to form what are known as *quality factors*:

$$Q_L = R_{AL}\sqrt{\frac{C_{AB}}{M_{AP}}}\, Q_{TS} = \frac{1}{R_{AE}+R_{AS}}\sqrt{\frac{M_{AC}}{C_{AS}}}$$

This notation allows for a simpler expression for the resulting transfer function [1]:

$$\frac{U_0}{V_{IN}} = G(s) = \frac{(s^3/\omega_0^4)}{(s/\omega_0)^4 + a_3(s/\omega_0)^3 + a_2(s/\omega_0)^2 + a_1(s/\omega_0) + 1}$$

where

$$a_1 = \frac{1}{Q_L\sqrt{h}} + \frac{\sqrt{h}}{Q_{TS}}\, a_2 = \frac{\alpha+1}{h} + h + \frac{1}{Q_L Q_{TS}}\, a_3 = \frac{1}{Q_{TS}\sqrt{h}} + \frac{\sqrt{h}}{Q_L}$$

Development of Low-Frequency Pressure Response

It can be shown [2] that for $ka < 1/2$, a loudspeaker behaves as a spherical source. Here, a represents the radius of the loudspeaker. For a 15" diameter loudspeaker in air, this low frequency limit is about 150 Hz. For smaller loudspeakers, this limit increases. This limit dominates the limit which ignores L_E, and is consistent with the limit that models Z_{RAD} by M_{A1}.

Within this limit, the loudspeaker emits a volume velocity U_0, as determined in the previous section. For a simple spherical source with volume velocity U_0, the far-field pressure is given by [1]:

$$p(r) \simeq j\omega\rho_0 U_0 \frac{e^{-jkr}}{4\pi r}$$

It is possible to simply let $r = 1$ for this analysis without loss of generality because distance is only a function of the surroundings, not the loudspeaker. Also, because the transfer function magnitude is of primary interest, the exponential term, which has a unity magnitude, is omitted. Hence, the pressure response of the system is given by [1]:

$$\frac{p}{V_{IN}} = \frac{\rho_0 s}{4\pi}\frac{U_0}{V_{IN}} = \frac{\rho_0 Bl}{4\pi S_D R_E M_A S} H(s)$$

Where $H(s) = sG(s)$. In the following sections, design methods will focus on $|H(s)|^2$ rather than $H(s)$, which is given by:

$$|H(s)|^2 = \frac{\Omega^8}{\Omega^8 + (a_3^2 - 2a_2)\,\Omega^6 + (a_2^2 + 2 - 2a_1a_3)\,\Omega^4 + (a_1^2 - 2a_2)\,\Omega^2 + 1}\,\Omega = \frac{\omega}{\omega_0}$$

This also implicitly ignores the constants in front of $|H(s)|$ since they simply scale the response and do not affect the shape of the frequency response curve.

Alignments

A popular way to determine the ideal parameters has been through the use of alignments. The concept of alignments is based upon filter theory. Filter development is a method of selecting the poles (and possibly zeros) of a transfer function to meet a particular design criterion. The criteria are the desired properties of a magnitude-squared transfer function, which in this case is $|H(s)|^2$. From any of the design criteria, the poles (and possibly zeros) of $|H(s)|^2$ are found, which can then be used to calculate the numerator and denominator. This is the "optimal" transfer function, which has coefficients that are matched to the parameters of $|H(s)|^2$ to compute the appropriate values that will yield a design that meets the criteria.

There are many different types of filter designs, each which have trade-offs associated with them. However, this design is limited because of the structure of $|H(s)|^2$. In particular, it has the structure of a fourth-order high-pass filter with all zeros at $s = 0$. Therefore, only those filter design methods which produce a low-pass filter with only poles will be acceptable methods to use. From the traditional set of algorithms, only Butterworth and Chebyshev low-pass filters have only poles. In addition, another type of filter called a quasi-Butterworth filter can also be used, which has similar properties to a Butterworth filter. These three algorithms are fairly simple, thus they are the most popular. When these low-pass filters are converted to high-pass filters, the $s \rightarrow 1/s$ transformation produces s^8 in the numerator.

More details regarding filter theory and these relationships can be found in numerous resources, including [5].

Butterworth Alignment

The Butterworth algorithm is designed to have a *maximally flat* pass band. Since the slope of a function corresponds to its derivatives, a flat function will have derivatives equal to zero. Since as flat of a pass band as possible is optimal, the ideal function will have as many derivatives equal to zero as possible at $s = 0$. Of course, if all derivatives were equal to zero, then the function would be a constant, which performs no filtering.

Often, it is better to examine what is called the *loss function*. Loss is the reciprocal of gain, thus

$$|\hat{H}(s)|^2 = \frac{1}{|H(s)|^2}$$

The loss function can be used to achieve the desired properties, then the desired gain function is recovered from the loss function.

Now, applying the desired Butterworth property of maximal pass-band flatness, the loss function is simply a polynomial with derivatives equal to zero at $s = 0$. At the same time, the original polynomial must be of degree eight (yielding a fourth-order function). However, derivatives one through seven can be equal to zero if [3]

$$|\hat{H}(\Omega)|^2 = 1 + \Omega^8 \Rightarrow |H(\Omega)|^2 = \frac{1}{1 + \Omega^8}$$

With the high-pass transformation $\Omega \rightarrow 1/\Omega$,

$$|H(\Omega)|^2 = \frac{\Omega^8}{\Omega^8 + 1}$$

It is convenient to define $\Omega = \omega / \omega_{3dB}$, since $\Omega = 1 \Rightarrow |H(s)|^2 = 0.5$ or -3 dB. This defintion allows the matching of coefficients for the $|H(s)|^2$ describing the loudspeaker response when $\omega_{3dB} = \omega_0$. From this matching, the following design equations are obtained [1]:

$$a_1 = a_3 = \sqrt{4 + 2\sqrt{2}} \quad a_2 = 2 + \sqrt{2}$$

Quasi-Butterworth Alignment

The quasi-Butterworth alignments do not have as well-defined of an algorithm when compared to the Butterworth alignment. The name "quasi-Butterworth" comes from the fact that the transfer functions for these responses appear similar to the Butterworth ones, with (in general) the addition of terms in the denominator. This will be illustrated below. While there are many types of quasi-Butterworth alignments, the simplest and most popular is the 3rd order alignment (QB3). The comparison of the QB3 magnitude-squared response against the 4th order Butterworth is shown below.

$$|H_{QB3}(\omega)|^2 = \frac{(\omega/\omega_{3dB})^8}{(\omega/\omega_{3dB})^8 + B^2(\omega/\omega_{3dB})^2 + 1} \quad |H_{B4}(\omega)|^2 = \frac{(\omega/\omega_{3dB})^8}{(\omega/\omega_{3dB})^8 + 1}$$

Notice that the case $B = 0$ is the Butterworth alignment. The reason that this QB alignment is called 3rd order is due to the fact that as B increases, the slope approaches 3 dec/dec instead of 4 dec/dec, as in 4th order Butterworth. This phenomenon can be seen in Figure 5.

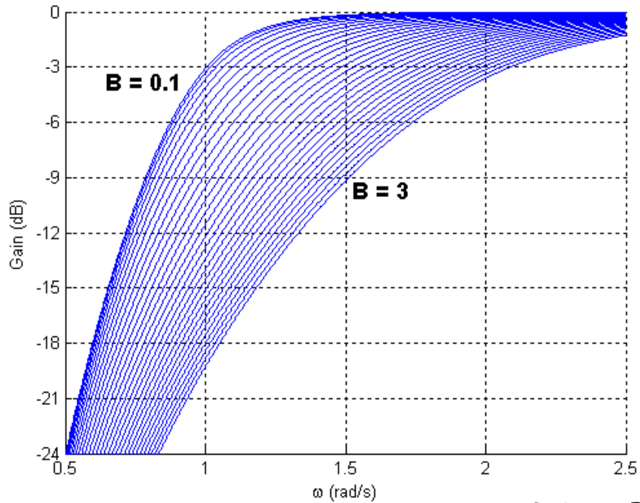


Figure 5: 3rd-Order Quasi-Butterworth Response for $0.1 \leq B \leq 3$

Equating the system response $|H(s)|^2$ with $|H_{QB3}(s)|^2$, the equations guiding the design can be found [1]:

$$B^2 = a_1^2 - 2a_2 \; a_2^2 + 2 = 2a_1a_3 \; a_3 = \sqrt{2a_2} \; a_2 > 2 + \sqrt{2}$$

Chebyshev Alignment

The Chebyshev algorithm is an alternative to the Butterworth algorithm. For the Chebyshev response, the maximally-flat passband restriction is abandoned. Now, a *ripple*, or fluctuation is allowed in the pass band. This allows a steeper transition or roll-off to occur. In this type of application, the low-frequency response of the loudspeaker can be extended beyond what can be achieved by Butterworth-type filters. An example plot of a Chebyshev high-pass response with 0.5 dB of ripple against a Butterworth high-pass response for the same ω_{3dB} is shown below.

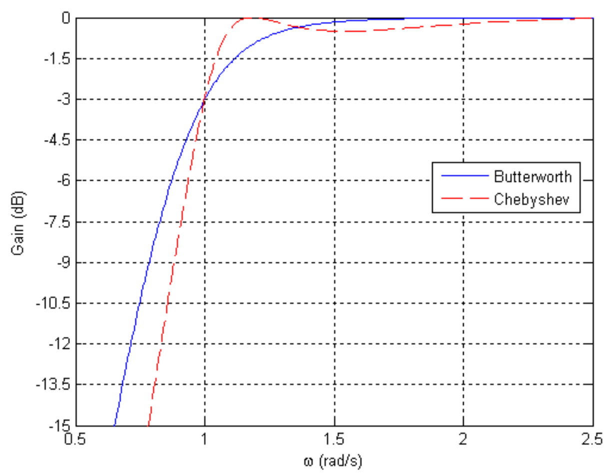


Figure 6: Chebyshev vs. Butterworth High-Pass Response.

The Chebyshev response is defined by [4]:

$$|\hat{H}(j\Omega)|^2 = 1 + \epsilon^2 C_n^2(\Omega)$$

$C_n(\Omega)$ is called the *Chebyshev polynomial* and is defined by [4]:

$$C_n(\Omega) = \begin{cases} \cos[n\cos^{-1}(\Omega)] & |\Omega| < 1 \\ \cosh[n\cosh^{-1}(\Omega)] & |\Omega| > 1 \end{cases}$$

Fortunately, Chebyshev polynomials satisfy a simple recursion formula [4]:

$$C_0(x) = 1 \quad C_1(x) = x \quad C_n(x) = 2xC_{n-1} - C_{n-2}$$

For more information on Chebyshev polynomials, see the Wolfram Mathworld: Chebyshev Polynomials page.

When applying the high-pass transformation to the 4th order form of $|\hat{H}(j\Omega)|^2$, the desired response has the form [1]:

$$|H(j\Omega)|^2 = \frac{1 + \epsilon^2}{1 + \epsilon^2 C_4^2(1/\Omega)}$$

The parameter ε determines the ripple. In particular, the magnitude of the ripple is $10\log[1 + \epsilon^2]$ dB and can be chosen by the designer, similar to *B* in the quasi-Butterworth case. Using the recursion formula for $C_n(x)$,

$$C_4\left(\frac{1}{\Omega}\right) = 8\left(\frac{1}{\Omega}\right)^4 - 8\left(\frac{1}{\Omega}\right)^2 + 1$$

Applying this equation to | $H(j\Omega)$ | ^{2} [1],

$$\Rightarrow |H(\Omega)|^2 = \frac{\frac{1+\epsilon^2}{64\epsilon^2}\Omega^8}{\frac{1+\epsilon^2}{64\epsilon^2}\Omega^8 + \frac{1}{4}\Omega^6 + \frac{5}{4}\Omega^4 - 2\Omega^2 + 1}$$

$$\Omega = \frac{\omega}{\omega_n} \quad \omega_n = \frac{\omega_{3dB}}{2}\sqrt{2 + \sqrt{2 + 2\sqrt{2 + \frac{1}{\epsilon^2}}}}$$

Thus, the design equations become [1]:

$$\omega_0 = \omega_n \sqrt[8]{\frac{64\epsilon^2}{1+\epsilon^2}} \quad k = \tanh\left[\frac{1}{4}\sinh^{-1}\left(\frac{1}{\epsilon}\right)\right] D = \frac{k^4 + 6k^2 + 1}{8}$$

$$a_1 = \frac{k\sqrt{4 + 2\sqrt{2}}}{\sqrt[4]{D}}, a_2 = \frac{1 + k^2(1 + \sqrt{2})}{\sqrt{D}} \quad a_3 = \frac{a_1}{\sqrt{D}}\left[1 - \frac{1 - k^2}{2\sqrt{2}}\right]$$

Choosing the Correct Alignment

With all the equations that have already been presented, the question naturally arises, “Which one should I choose?” Notice that the coefficients a_1, a_2, and a_3 are not simply related to the parameters of the system response. Certain combinations of parameters may indeed invalidate one or more of the alignments because they cannot realize the necessary coefficients. With this in mind, general guidelines have been developed to guide the selection of the appropriate alignment. This is very useful if one is designing an enclosure to suit a particular transducer that cannot be changed.

The general guideline for the Butterworth alignment focuses on Q_L and Q_{TS}. Since the three coefficients a_1, a_2, and a_3 are a function of Q_L, Q_{TS}, h, and α, fixing one of these parameters yields three equations that uniquely determine the other three. In the case where a particular transducer is already given, Q_{TS} is essentially fixed. If the desired parameters of the enclosure are already known, then Q_L is a better starting point.

In the case that the rigid requirements of the Butterworth alignment cannot be satisfied, the quasi-Butterworth alignment is often applied when Q_{TS} is not large enough.. The addition of another parameter, B, allows more flexibility in the design. For Q_{TS} values that are too large for the Butterworth alignment, the Chebyshev alignment is typically chosen. However, the steep transition of the Chebyshev alignment may also be utilized to attempt to extend the bass response of the loudspeaker in the case where the transducer properties can be changed.

In addition to these three popular alignments, research continues in the area of developing new algorithms that can manipulate the low-frequency response of the bass-reflex enclosure. For example, a 5th order quasi-Butterworth alignment has been developed [6]. Another example [7] applies root-locus techniques to achieve results. In the modern age of high-powered computing, other researchers have focused their efforts in creating computerized optimization algorithms that can be modified to achieve a flatter response with sharp roll-off or introduce quasi-ripples which provide a boost in sub-bass frequencies [8].

References

[1] Leach, W. Marshall, Jr. *Introduction to Electroacoustics and Audio Amplifier Design*. 2nd ed. Kendall/Hunt, Dubuque, IA. 2001.
[2] Beranek, L. L. *Acoustics*. 2nd ed. Acoustical Society of America, Woodbridge, NY. 1993.
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[4] DeCarlo, Raymond A. “The Chebyshev Approximation.” Notes from ECE 445. Purdue University. 2004.
[5] VanValkenburg, M. E. *Analog Filter Design*. Holt, Rinehart and Winston, Inc. Chicago, IL. 1982.
[6] Kreutz, Joseph and Panzer, Joerg. "Derivation of the Quasi-Butterworth 5 Alignments." *Journal of the Audio Engineering Society*. Vol. 42, No. 5, May 1994.
[7] Rutt, Thomas E. "Root-Locus Technique for Vented-Box Loudspeaker Design." *Journal of the Audio Engineering Society*. Vol. 33, No. 9, September 1985.
[8] Simeonov, Lubomir B. and Shopova-Simeonova, Elena. "Passive-Radiator Loudspeaker System Design Software Including Optimization Algorithm." *Journal of the Audio Engineering Society*. Vol. 47, No. 4, April 1999.

Appendix A: Equivalent Circuit Parameters

Name	Electrical Equivalent	Mechanical Equivalent	Acoustical Equivalent
Voice-Coil Resistance	R_E	$R_{ME} = \frac{(Bl)^2}{R_E}$	$R_{AE} = \frac{(Bl)^2}{R_E S_D^2}$
Driver (Speaker) Mass	See C_{MEC}	M_{MD}	$M_{AD} = \frac{M_{MD}}{S_D^2}$
Driver (Speaker) Suspension Compliance	$L_{CES} = (Bl)^2 C_{MS}$	C_{MS}	$C_{AS} = S_D^2 C_{MS}$
Driver (Speaker) Suspension Resistance	$R_{ES} = \frac{(Bl)^2}{R_{MS}}$	R_{MS}	$R_{AS} = \frac{R_{MS}}{S_D^2}$
Enclosure Compliance	$L_{CEB} = \frac{(Bl)^2 C_{AB}}{S_D^2}$	$C_{MB} = \frac{C_{AB}}{S_D^2}$	C_{AB}
Enclosure Air-Leak Losses	$R_{EL} = \frac{(Bl)^2}{S_D^2 R_{AL}}$	$R_{ML} = S_D^2 R_{AL}$	R_{AL}
Acoustic Mass of Port	$C_{MEP} = \frac{S_D^2 M_{AP}}{(Bl)^2}$	$M_{MP} = S_D^2 M_{AP}$	M_{AP}
Enclosure Mass Load	See C_{MEC}	See M_{MC}	M_{AB}
Low-Frequency Radiation Mass Load	See C_{MEC}	See M_{MC}	M_{A1}
Combination Mass Load	$C_{MEC} = \frac{S_D^2 M_{AC}}{(Bl)^2} = \frac{S_D^2(M_{AB} + M_{A1}) + M_{MD}}{(Bl)^2}$	$M_{MC} = S_D^2(M_{AB} + M_{A1}) + M_{MD}$	$M_{AC} = M_{AD} + M_{AB} + M_{A1} = \frac{M_{MD}}{S_D^2} + M_{AB} + M_{A1}$

Appendix B: Enclosure Parameter Formulas

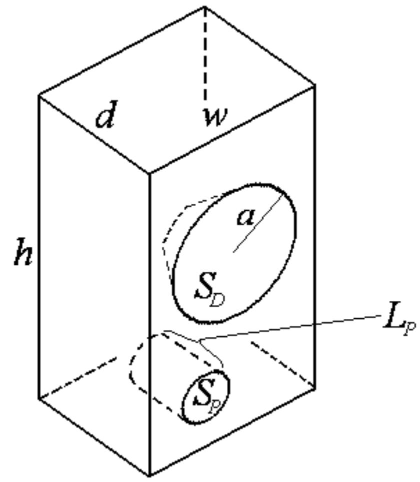

Figure 7: Important dimensions of bass-reflex enclosure.

Based on these dimensions [1],

$$C_{AB} = \frac{V_{AB}}{\rho_0 c_0^2} \qquad M_{AB} = \frac{B\rho_{eff}}{\pi a}$$

$$B = \frac{d}{3}\left(\frac{S_D}{S_B}\right)^2 \sqrt{\frac{\pi}{S_D}} + \frac{8}{3\pi}\left[1 - \frac{S_D}{S_B}\right] \qquad \rho_0 \le \rho_{eff} \le \rho_0\left(1 - \frac{V_{fill}}{V_B}\right) + \rho_{fill}\frac{V_{fill}}{V_B}$$

$$V_{AB} = V_B\left[1 - \frac{V_{fill}}{V_B}\right]\left[1 + \frac{\gamma - 1}{1 + \gamma\left(\frac{V_B}{V_{fill}} - 1\right)\frac{\rho_0 c_{air}}{\rho_{fill} c_{fill}}}\right]$$

$V_B = hwd$ (inside enclosure volume)

$S_B = wh$ (inside area of the side the speaker is mounted on)

c_{air} = specific heat of air at constant volume

c_{fill} = specific heat of filling at constant volume ($V_{fill\,ing}$)

ρ_0 = mean density of air (about 1.3 kg/m^3)

ρ_{fill} = density of filling

γ = ratio of specific heats for air (1.4)

c_0 = speed of sound in air (about 344 m/s)

ρ_{eff} = effective density of enclosure. If little or no filling (acceptable assumption in a bass-reflex system but not for sealed enclosures), $\rho_{eff} \approx \rho_0$